B.E AERONAUTICAL ENGINEERING
Choice Based Credit System (CBCS) and Outcome Based Education (OBE)
SEMESTER - VII

COMPUTATIONAL FLUID DYNAMICS

Course Code	**18AE72**	CIE Marks	40
Teaching Hours/Week (L:T:P)	(3:0:0)	SEE Marks	60
Credits	03	Exam Hours	03

Course Learning Objectives:

- Know the basic equations of fluid dynamics, boundary layer and discretization.
- Understand the source and vortex panel method.
- Know about FDM, FVM and FEM.

Module-1

Introduction: CFD Applications. Need for Parallel Computers in CFD algorithms. Models of flows. Substantial derivative, Divergence of velocity. Continuity, Momentum, and Energy Equations-Derivation in various forms. Integral versus Differential form of equations. Comments on governing equations. Physical boundary conditions. Forms of equations especially suitable for CFD work. Shock capturing, and shock fitting.

Module-2

Mathematical Behaviour of Partial Differential Equations: Classification of partial differential equations. Cramer Rule and Eigen value methods for classification. Hyperbolic, parabolic, and elliptic forms of equations. Impact of classification on physical and computational fluid dynamics. Case studies: steady inviscid supersonic flow, unsteady inviscid flow, steady boundary layer flow, and unsteady thermal conduction, steady subsonic inviscid flow.

Module-3

Grid Generation and Adaptive Grids: Need for grid generation and Body-fitted coordinate system. Structured Grids-essential features. Structured Grid generation techniques- algebraic and numerical methods. Unstructured Grids-essential features. Unstructured Grid generation techniques- Delaunay-Voronoi diagram, advancing front method. Surface grid generation, multi-block grid generation, and meshless methods. Grid quality and adaptive grids. Structured grids adaptive methods and unstructured grids adaptive methods.

Module-4

Discretisation & Transformation:
Discretisation: Finite differences methods, and difference equations. Explicit and Implicit approaches. Unsteady Problem -Explicit versus Implicit Scheme. Errors and stability analysis. Time marching and space marching. Reflection boundary condition. Relaxation techniques. Alternating direction implicit method. Successive over relaxation/under relaxation. Second order Lax-Wendroff method, mid-point Leap frog method, upwind scheme, numerical viscosity, and artificial viscosity.
Transformation: Transformation of governing partial differential equations from physical domain to computational domain. Matrices and Jacobians of transformation. Example of transformation. Generic form of the Governing flow equations in Strong Conservative form in the Transformed Space.

Module-5

Finite Volume Technique and Some Applications: Spatial discretisation- cell centered and cell vertex techniques (overlapping control volume, duel control volume). Temporal discretisation- Explicit time stepping, and implicit time stepping. Time step calculation. Upwind scheme and high resolution scheme. Flux vector splitting, approximate factorisation. Artificial dissipation and flux limiters. Unsteady flows and heat conduction problems. Upwind biasing.

Course Outcomes: At the end of the course the student will be able to:

1. CO1 :Differentiate the FDM, FVM and FEM
2. CO2: Perform the flow, structural and thermal analysis.
3. CO3: Utilize the discretization methods according to the application.

PROFESSOR & HEAD
Department of Aeronautical Engineering
S.J.C. Institute of Technology
CHICKBALLAPUR-562101

Question paper pattern:

- The question paper will have ten full questions carrying equal marks.
- Each full question will be for 20 marks.
- There will be two full questions (with a maximum of four sub- questions) from each module.
- Each full question will have sub- question covering all the topics under a module.
- The students will have to answer five full questions, selecting one full question from each module.

Sl. No.	Title of the Book	Name of the Author/s	Name of the Publisher	Edition and Year
Textbook/s				
1	Applied Computational Fluid Dynamics	Gupta S.C	Wiley, India	2019
2	Computational Fluid Dynamics	John D. Anderson	McGraw Hill	2013
Reference Books				
1	Computational Fluid Dynamics-An Introduction	John F. Wendt	Springer	3rd Edition, 2013
2	Numerical Computation of Internal and External Flows	Charles Hirsch	Elsevier	1st edition,2007
3	Computational Fluid Dynamics for Engineers	Klaus A Hoffmann and SteveT. Chiang		1993
4	Fundamentals of CFD	Tapan K. Sengupta	Universities Press	2004

Dept - AERO

CBCS SCHEME

USN | | | | | | | | | | | 15AE72

Seventh Semester B.E. Degree Examination, Aug./Sept.2020
Computational Fluid Dynamics

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Derive momentum equation for small fluid element fixed in space and for small element moving in space, with viscous terms. **(10 Marks)**

b. Show that substantial derivative $\rho \frac{Du}{Dt}$ occurring in non-conservative form of momentum equation can be written in the following way that is representative of conservative form $\frac{\partial(\rho u)}{\partial t} + \nabla.(\rho u V)$ **(06 Marks)**

OR

2 a. What are the various boundary conditions? **(06 Marks)**

b. What are CFD ideas to understand? **(10 Marks)**

Module-2

3 a. Through the Cramer rule determine the slopes of characteristic lines for potential 2-D flow equation $\left(1 - M_\infty^2\right)\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, where u and v are perturbation velocities in the flow. M_∞ is free stream Mach number. **(12 Marks)**

b. Explain how steady boundary layer flow can be governed by Parabolic equations. **(04 Marks)**

OR

4 Explain the different mathematical behavior of CFD equations that reflects different physical behavior of flow field. Give an example of each case. **(16 Marks)**

Module-3

5 a. Describe Hermite Polynomial Interpolation. **(08 Marks)**

b. Develop a cubic Hermite Polynomial for following function
$f(x) = x^4 + x^3 + x^2 + x + 1$ **(08 Marks)**

OR

6 a. Explain elliptic grid generation technique. **(08 Marks)**

b. Describe the following for structured adaptive grid generation:
(i) Control function approach
(ii) Variational methods **(08 Marks)**

Module-4

7 a. Describe the following :
(i) Upwind differencing
(ii) Midpoint leap frog differencing techniques
(iii) Reflection Boundary condition. **(09 Marks)**

b. Use an explicit numerical method to solve the heat conduction equation

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2}$$

Boundary conditions : $T(0, t) = T(1, t) = 0 \quad (t \geq 0)$; $T(x, 0) = \sin(\pi x) \quad (0 \leq x \leq 1)$
Both ends held at zero temperature given initial temperature distribution.
Use the following parameters.
$\Delta t = 0.1$ $\Delta x = 0.25$ $a = 0.1$; Carry out iterations till 0.3 sec. **(07 Marks)**

OR

8 a. Consider the following transformation for accomplishing grid stretching :

$$\xi = x,\ \eta = \ln(y + 1)$$

What happens to governing flow equations in both the physical and computational plane with this transformation? Show this with an example of 2-D continuity equation for compressible flow through matrices technique for transformation of grids. **(10 Marks)**

b. Explain the above with Inverse Transformation through use of Jacobean. **(06 Marks)**

Module-5

9 Write short notes on following :

a. Numerical viscosity **(04 Marks)**

b. Flux vector splitting **(06 Marks)**

c. Approximate Factorisation **(06 Marks)**

OR

10 Explain the following :

a. Artificial viscosity **(04 Marks)**

b. Finite volume solution to diffusion problem below.

$$\frac{d}{dx}\left(k\frac{\partial T}{\partial x}\right) = 0$$ **(05 Marks)**

c. Finite volume solution to convection and diffusion problem below.

$$\frac{d}{dx}\left(k\frac{\partial T}{\partial x}\right) - \frac{d(\rho u T)}{dx} = 0$$ **(07 Marks)**

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USN 1 S J 1 6 A E 0 2 2 — 15AE72

Seventh Semester B.E. Degree Examination, Dec.2019/Jan.2020
Computational Fluid Dynamics

Time: 3 hrs. Max. Marks: 80

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. With the help of neat sketches, explain the different models of the flow. (06 Marks)

b. Derive the momentum equation considering an infinitesimally small fluid element moving with the flow, for an unsteady. Three dimensional, compressible and various flow with usual notations. (10 Marks)

OR

2 a. Explain the importance of CFD in modern study and the different architectures used in CFD. (08 Marks)

b. Derive an expression for divergence of velocity with usual notation and explain its physical meaning. (08 Marks)

Module-2

3 a. Explain the different mathematical behavior of CFD equation that reflects different physied behavior of flow. Given an example for each case. (08 Marks)

b. Describe the external features of hyperbolic equation and explain its impact on physical behavior of CFD problems. (08 Marks)

OR

4 a. Consider the irrigational two dimensional, inviscid steady flow of a compressible gas. If the flow field is only slight perturbed from the freestream conditions such as the flow over as thin body as small angles of attack and if the freestream mach number is either subsonic or supersonic (but not transonic or hypersonic) the governing conformity, momentum and energy equation can be reduced to the systems. Find the roots of equations involved in such kind of flow problem using Cramer's rule. (08 Marks)

b. Explain the following with relevant sketches :

i) Parabolised viscous flow

ii) Unsteady in viscous flow. (08 Marks)

Module-3

5 a. With the help of relevant sketch explain the elliptic grid generation. (08 Marks)

b. Define grid quality. List the measures of quality and explain in detail. (08 Marks)

OR

6 a. List the advantages and disadvantages of structured and unstructured grids. Explain in brief. (08 Marks)

b. Write short notes on : i) Adaptive grids ii) Meshless grids. (08 Marks)

Module-4

7 a. Demonstrate the explicit and implicit approach of solving CFD problems. List their advantages and disadvantages. **(08 Marks)**

b. For an inviscid, incompressible, two-dimensional irrotational flow in a Cartesian space explain the steps involving in numerical solving by relaxation technique with suitable expressions. **(08 Marks)**

OR

8 a. Explain numerical and artificial viscosity with suitable expressions. **(08 Marks)**

b. With suitable expressions, demonstrate the transformation of governing partial differential equations from physical domain to computational domain. **(08 Marks)**

Module-5

9 a. Write short notes on :

i) Cell-centered technique

ii) Cell-vertex technique. **(10 Marks)**

b. With suitable expression explain explicit time stepping scheme. **(06 Marks)**

OR

10 Describe the following finite volume techniques with their applications :

i) Flux vector splitting

ii) Spatial discritization. **(16 Marks)**

* * * * *

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SJC INSTITUTE OF TECHNOLOGY

Estd: 1986

Chickballapur – 562 101

Department of Aeronautical Engineering

QUESTION BANK

SUBJECT TITLE	COMPUTATIONAL FLUID DYNAMICS		
SUBJECT TYPE	CORE / ~~ELECTIVE~~		
SUBJECT CODE	17AE82		
ACADEMIC YEAR	2021-22 (ODD SEMESTER)	BATCH	2018-2022
SCHEME	CBCS scheme (Effective from the academic year 2015 -2016)		
SEMESTER	VII		
FACULTY NAME and DESIGNATION	Prof. Deepa M S		

Module -1

Q. No.	*Questions*	*Bloom's LL*	*COs*
1	Explain the different computer architectures used in CFD. Explain the importance of CFD in modern study and the solution of problems in Fluid Mechanics.	L2	CO1
2	With the help of a neat sketch explain the method of solving flow field over a supersonic blunt-nosed body	L2	CO1
3	With the help of neat sketches explain the different models of the flow.	L2	CO1
4	Derive an expression for Substantial Derivative with usual notations	L3	CO1
5	Derive an expression for Time Rate of change following a moving fluid element	L3	CO1
6	Explain the models of the flows to obtain Conservation and Non-conservation form of Equations	L2	CO1
7	Derive an expression for Divergence of Velocity with usual notations	L3	CO1
8	With the help of a neat sketch explain the models of Finite Control Volume.	L2	CO1
9	Derive the Momentum Equation considering an infinitesimally small fluid element moving with the flow, for unsteady, three-dimensional, compressible and viscous flow with usual notations	L3	CO1
10	Derive the Energy Equation considering an infinitesimally small fluid element moving with the flow, with usual notations	L3	CO1

Module -2			
Q. No.	*Questions*	*Bloom's LL*	*COs*
1	Explain the classification of Quasi-Linear Partial Differential Equations	L2	CO2
2	Apply Eigen Value method to a Quasi-linear partial differential equation for the mathematical classification as elliptic, parabolic and hyperbolic.	L3	CO2
3	Apply Cramer Rule to a Quasi-linear partial differential equation for the mathematical classification as elliptic, parabolic and hyperbolic.	L3	CO2
4	Explain the different mathematical behavior of CFD equations that reflects different physical behavior of flow: give an example in each case.	L2	CO2
5	Assuming a system of quasi-linear equations, with the help of a characteristic curve at a point p(x,y), classify the different types of PDE's stating example for each type.	L3	CO2
6	Determine the type of PDE for the following equation: Also if a variable Φ is introduced such that u = and v = Determine the nature of PDE.	L4	CO2
7	Describe the essential features of hyperbolic equation impact on physical behavior of CFD problems.	L3	CO2
8	Explain the following with relevant sketches i) Steady Inviscid Supersonic Flow ii) Unsteady Inviscid Flow	L3	CO2
9	Describe the essential features of parabolic equation impact on physical behavior of CFD problems.	L3	CO2
10	Explain the following with relevant sketches i) Steady Boundary Layer Flows ii) Parabolized Viscous Flow iii) Unsteady Thermal Conduction	L3	CO2
11	For the one dimensional unsteady thermal conduction through a semi-infinite fluid, write the governing equation, boundary conditions and plot the typical solution characteristics.	L3	CO2
12	Consider the irrotational, 2-D steady flow of a compressible gas. The flow field is slightly perturbed from free stream like flow over a thin profile. Find the roots of equations involved in such kind of flow problem, using Eigen method.	L4	CO2

Module -3			
Q. No.	*Questions*	*Bloom's LL*	*COs*
1.	Define grid generation and describe the importance of boundary fitted co-ordinate system in CFD.	L2	CO3
2.	With the help of relevant sketches explain boundary fitted co-ordinate system	L2	CO3
3.	Explain the boundary fitted coordinate system for the divergent duct.	L2	CO3
4.	List the features of Structured grids? Explain the different methods of structured grid generation with suitable sketches.	L2	CO3
5.	Explain the following with relevant sketches i) Algebraic Methods ii) Differential Equation Technique or Numerical Method	L2	CO3
6.	List the features of Unstructured grids? Explain the different methods of unstructured grid generation with suitable sketches.	L2	CO3
7.	Explain the following with relevant sketches i) Point Insertion Schemes or Delaunay-Voronoi Diagram ii) Advancing Front Methods	L2	CO3
8.	Discuss the need of grid generation of grid generation in CFD and summarize the following methods of grid generation. i) Surface Grid Generation ii) Multi Block Grid Generation iii) Meshless Methods	L2	CO3
9.	Discuss the following i) Striving for quality ii) Grid design guidelines and total cell count	L2	CO3
10.	With the help of relevant sketches explain the adaptive grid generation.	L2	CO3
11.	Differentiate Structured and Unstructured Adaptive grids with suitable sketches.	L2	CO3
12.	List the essential properties of grids	L2	CO3
13.	Explain the importance of various coordinate systems in grid generation	L2	CO3
14.	Consider a Trapezoid in (x,y) plane as shown in fig. Generate a mesh corresponding to (ξ,η) coordinates at 0.2 units apart. Take four points of Trapezoid as A(0,0), B(20,0), C(20,10) and D(0,5)	L3,L4	CO3

Module -4			
Q. No.	**Questions**	**Bloom's LL**	**COs**
1.	Summarize the essence of discretization in CFD	L2	CO4
2.	Derive the Taylor Series approach for the construction of finite Difference Equation	L3	CO4
3.	With the help of neat grids explain the graphical concept of finite difference modules and list the pros and cons of higher order accuracy	L3	CO4
4.	Illustrate a portion of a boundary and explain what happens at the boundary. Also describe the reflection boundary conditions	L3	CO4
5.	Derive a difference equation for an unsteady one dimensional heat conduction equation with thermal diffusivity with usual notations.	L3	CO4
6.	Demonstrate the Explicit and Implicit approaches of solving CFD. List their advantages and disadvantages	L3	CO4
7.	Explain the different types of errors involved in solving finite difference equations and summarize the stability analysis	L2	CO4
8.	Consider the viscous flow of air over a flat plate. At a given station in the flow direction, the variation of the flow velocity u in the direction perpendicular to the plate (the y direction) is given by the expression $u = 1582(1-e^{-y/L})$ where L is the characteristics length – 0.0988 m The units of u is m/s. the viscosity co-efficient $\mu = 1.7894 \times 10^{-5}$ W/m-s. the above equation is used to provide the values of u at discrete grid points equally spaced in the y-direction Y(m) u(m/s) 0 0 0.03 45.88 0.006 87.407 0.009 124.977 The values of u listed above are discrete values at the distance grid points located at y = 0, 0.003, 0.006, 0.009 in the same nature as would be obtained from a numerical finite-difference solution of the flow field. Using these discrete values calculate the shear stress at the wall τ_w in three different ways namely i) First order one-sided difference ii) Second order one-sided difference iii) Third order one-sided difference Finally compare these calculated finite-difference results with the exact value of τ_w as specified by the equation above	L4	CO4
9.	If = f(x,y), solve the flow domain using FVM via FDM by considering a 3x2 unit square mesh as shown in Fig. The various boundary values are u1, u2, u3, u6, u5, u9, u12 = 0 nad u4 = 8, u7 = 32, u11 = 18, f5 = 8, f8 = 20. Obtain the values of u5 and u8.	L4	CO4

Module -5			
Q. No.	*Questions*	*Bloom's LL*	*COs*
1.	With the help of neat grids and equations explain i) Relaxation Technique ii) Alternating Direct Implicit (ADI) Method iii) Successive Over relaxation and Under Relaxation iv) Second Order Lax-Wendorff Method v) Mid-Point Leap Frog Method vi) Upwind Scheme	L3	CO5
2.	With the help of relevant sketches illustrate Numerical Dissipation, Numerical Diffusion and Numerical and Artificial Viscosity	L2	CO5
3.	Explain Lax-Wendroff technique for Time marching (VTU, Dec 2009-2010 – 12 Marks)	L2	CO5
4.	What is Upwind Scheme? Explain in brief (VTU, Dec 2009-2010 – 8 Marks)	L2	CO5
5.	What is alternating Director Implicit (ADI) technique? Explain(VTU, Dec 2010 – 10 Marks)	L2	CO5
6.	What is successive over-relaxation and under-relaxation (VTU, June/July 2011 – 8 Marks)	L2	CO5
7.	For a 2D, unsteady heat conduction in a Cartesian space explain the steps involved in numerical solving by DI technique with suitable expressions	L3	CO5
8.	Derive the expression for amplification factor and stability requirement for a numerical solution using Lax-Wendroff technique using wave equation with one-step scheme.	L3	CO5
9.	Write short notes on: i) Time and space marching in CFD ii) Various general schemes of solution of PDEs numerically iii) Upwind schemes in CFD (VTU, Dec 2011 – 20 Marks)	L2	CO5

Note:

1. Questions shall be framed by consolidating comprehensively from the following sources
    - Exercise problems of text books/ references
    - Previous year question VTU exam Question paper. (Mark the year/exam beside the question)
    - Questions by Experts during Interview/Academic Audit
    - Internet sources/ other Universities examination question papers.
    - Own / experience.
    - Gate questions mentioning the year.
2. Questions shall follow all the Bloom's learning levels with appropriate action verbs
3. There shall be a total of 50 questions considering10 questions from each module, of which, 3 questions each at L1 and L2, 2 questions at L3, 1 question each at L4 and L5/L6.
4. Ensure the coverage of all Cos.

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SJC INSTITUTE OF TECHNOLOGY

Estd: 1986

Chickballapur – 562 101

Department of Aeronautical Engineering
ASSIGNMENT

SUBJECT TITLE	COMPUTATIONAL FLUID DYNAMICS		
SUBJECT TYPE	CORE / ~~ELECTIVE~~		
SUBJECT CODE	17AE82		
ACADEMIC YEAR	2021-22 (ODD SEMESTER)	BATCH	2018-2022
SCHEME	CBCS scheme (Effective from the academic year 2016 -2017)		
SEMESTER	VII		
FACULTY NAME and DESIGNATION	Prof. Deepa M S		

Module -1 TO Module-5

Q. No.	*Questions MINI PROJECT TOPICS*	*Bloom's LL*	*COs*
1.	Modelling of 2-D Incompressible and Inviscid flow over Cambered Airfoil (NACA 2415) and plotting of pressure distribution and velocity vectors for Subsonic Mach number using ANSYS Workbench	L5 & L6	CO1-CO5
2.	Modelling of 2-D Incompressible and Inviscid flow over Cambered Airfoil (NACA 2415) and plotting of pressure distribution and velocity vectors for Supersonic Mach number using ANSYS Workbench	L5 & L6	CO1-CO5
3.	Modelling of 2-D Incompressible and Inviscid flow over Flat Plate of thickness 2 mm and 10 cm long and plotting of pressure distribution and velocity vectors for Subsonic Mach number using ANSYS Workbench	L5 & L6	CO1-CO5
4.	Modelling of 2-D Incompressible and Inviscid flow over Flat Plate of thickness 2 mm and 10 cm long and plotting of pressure distribution and velocity vectors for Supersonic Mach number	L5 & L6	CO1-CO5
5.	Modelling of 2-D Incompressible and Inviscid flow over Symmetric Airfoil (NACA 0015) and plotting of pressure distribution and velocity vectors for Subsonic Mach number	L5 & L6	CO1-CO5
6.	Modelling of 2-D Incompressible and Inviscid flow over a cylinder of 10 cm diameter and plotting of pressure distribution and velocity vectors for Supersonic Mach number	L5 & L6	CO1-CO5
7.	Modelling of 2-D Incompressible and Inviscid flow over a wedge of sides 10.5 and height 8.5 cm and plotting of pressure distribution and velocity vectors for Subsonic Mach number	L5 & L6	CO1-CO5
8.	Modelling of 2-D Incompressible and Inviscid flow over Symmetric Airfoil (NACA 0015) and plotting of pressure distribution and velocity vectors for Subsonic Mach number	L5 & L6	CO1-CO5

9.	Modelling of 2-D Incompressible and Inviscid flow over a wedge of sides 10.5 and height 8.5 cm and plotting of pressure distribution and velocity vectors for Subsonic Mach number	L5 & L6	CO1-CO5
10.	Modelling of 2-D Incompressible and Inviscid flow over a cylinder of 10 cm diameter and plotting of pressure distribution and velocity vectors for Subsonic Mach number	L5 & L6	CO1-CO5

Module -2

Q. No.	*Questions*	*Blooms LL*	*COs*
1	Determine the type of PDE for the following equation: Also if a variable Φ is introduced such that u = and v = Determine the nature of PDE.	L4	CO2
	Consider the irrotational, 2-D steady flow of a compressible gas. The flow field is slightly perturbed from free stream like flow over a thin profile. Find the roots of equations involved in such kind of flow problem, using Eigen method.	L4	CO2

Module -3

Q. No.	*Questions*	***Bloom's LL***	***COs***
4	Consider a Trapezoid in (x,y) plane as shown in fig. Generate a mesh corresponding to (ξ,η) coordinates at 0.2 units apart. Take four points of Trapezoid as A(0,0), B(20,0), C(20,10) and D(0,5)	L3,L4	CO3

Module -4

Q. No.	*Questions*	***Bloom's LL***	***COs***
1	Consider the viscous flow of air over a flat plate. At a given station in the flow direction, the variation of the flow velocity u in the direction perpendicular to the plate (the y direction) is given by the expression $u = 1582(1-e^{-y/L})$ where L is the characteristics length – 0.0988 m The units of u is m/s. the viscosity co-efficient $\mu = 1.7894 \times 10^{-5}$ W/m-s. the above equation is used to provide the values of u at discrete grid points	L4	CO4

<table>
<tr><td></td><td>equally spaced in the y-direction
<table><tr><th>Y(m)</th><th>u(m/s)</th></tr><tr><td>0</td><td>0</td></tr><tr><td>0.03</td><td>45.88</td></tr><tr><td>0.006</td><td>87.407</td></tr><tr><td>0.009</td><td>124.977</td></tr></table>The values of u listed above are discrete values at the distance grid points located at y = 0, 0.003, 0.006, 0.009 in the same nature as would be obtained from a numerical finite-difference solution of the flow field.
Using these discrete values calculate the shear stress at the wall τ_w in three different ways namely
i) First order one-sided difference
ii) Second order one-sided difference
iii) Third order one-sided difference
Finally compare these calculated finite-difference results with the exact value of τ_w as specified by the equation above</td><td></td><td></td></tr>
<tr><td></td><td>If = f(x,y), solve the flow domain using FVM via FDM by considering a 3x2 unit square mesh as shown in Fig. The various boundary values are u1, u2, u3, u6, u5, u9, u12 = 0 nad u4 = 8, u7 = 32, u11 = 18, f5 = 8, f8 = 20. Obtain the values of u5 and u8.</td><td>L4</td><td>CO4</td></tr>
</table>

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Estd: 1986

Chickballapur – 562 101

Department of Aeronautical Engineering

LESSON PLAN

SUBJECT TITLE	COMPUTATIONAL FLUID DYNAMICS		
SUBJECT TYPE	CORE / ~~ELECTIVE~~		
SUBJECT CODE	18AE72		
ACADEMIC YEAR	2021-22 (ODD SEMESTER)	BATCH	2018-2022
SCHEME	CBCS scheme (Effective from the academic year 2016 -2017)		
SEMESTER & SECTION	VII		
IA MARKS	40	*EXAM MARKS*	100
NUMBER OF LECTURE HOURs/WEEK	3	*TOTAL NUMBER OF LECTURE HOURS*	50
FACULTY NAME	Prof. Deepa M S	*NO. OF TIMES HANDLED*	Second time

COURSE LEARNING OBJECTIVES: This course will enable students to

1. Know the basic equations of fluid dynamics, boundary layer and discretization
2. Understand the source and vortex panel method
3. Know about FDM,FVM and FEM.

Course Outcomes: At the end of this course, students are able to:

CO1	Describe the basics of CFD, its governing equations of various flow models in differential and integral form. (L3)
CO2	Compare the physics of the flow with the mathematical behavior of partial differential equations. (L3)
CO3	Identify and compute the suitable grid generation and transformation techniques for a given problem. (L4)
CO4	Illustrate the fundamentals of discretization in FVM, FDM and FEM techniques. (L3)
CO5	Interpret different schemes and their stability in simple CFD applications. (L4)

CO-PO MATRIX

COURSE OUTCOMES	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2
CO1	3	3	2	2	2	-	-	-	2	1	-	3	3	2
CO2	3	3	3	3	2	-	-	-	2	1	-	3	3	2
CO3	3	2	1	2	1	-	-	-	1	1	-	2	3	3
CO4	3	3	3	2	1	-	-	-	1	1	-	2	3	3
CO5	3	3	3	2	1	-	-	-	1	1	-	2	3	3

Note:

Justification of CO-PO mapping

- Assignments from NPTEL in the form of quiz and Problems will be given as Assignment 1 and 2 respectively.
- A Mini Project covering all the COs and PSOs will be given as Assignment 3 to a group of 6-7 students

DELIVERY PLAN WITH DETAILS

MODULE – 1

Lecture #	Topic	Mode of Delivery (PlsTick √) 1	2	3	4	Date of Delivery	COs Covered
1.	Review of all the basics of mechanics of fluid and Aerodynamics	✓				} 4/10/2024	CO1
2.	**Introduction:** CFD Applications	✓					CO1
3.	Need for Parallel Computers in CFD algorithms. Models of flows.	✓				5/10/2024	CO1
4.	Substantial derivative, Divergence of velocity	✓				11/10/2024	CO1
5.	Continuity Equation-Derivation in various forms	✓				} 18/10/2024	CO1
6.	Continuity Equation-Derivation in various forms	✓				21/10/2024	CO1
7.	Momentum Equation- Derivation in various forms	✓				23/10/2024	CO1
8.	Energy Equations-.	✓					
9.	Integral versus Differential form of equations. Comments on governing equations	✓				25/10/2024	CO1
10.	Forms of equations especially suitable for CFD work. Shock capturing, and shock fitting	✓				27/10/2024	CO1

Textbook :	CFD by Anderson	#HOURS	Allotted	Taken
Signatures	Faculty: [signature] 15/11/2024		10	10
	HoD:			
Remarks	Module I Completed as per Schedule			

MODULE – 2

Lecture #	Topic	Mode of Delivery (PlsTick √) 1	2	3	4	Date of Delivery	COs Covered
11.	**Mathematical Behaviour of Partial Differential Equations:** Classification of partial differential equations.					28/10/2024	CO2
12.	Cramer Rule and Eigen value methods for classification					8/11/2024	CO2
13.	Hyperbolic forms of equations					9/11/2024	CO2
14.	Parabolic forms of equations					10/11/2024	CO2
15.	Elliptic forms of equations					11/11/2024	CO2
16.	Impact of classification on physical and computational fluid dynamics.					15/11/2024	CO2
17.	Case studies: steady inviscid supersonic flow, unsteady inviscid flow.					} 16/11/2024	CO2
18.	Steady boundary layer flow					17	CO2
19.	Steady subsonic inviscid flow					17/11/2024	CO2
20.	Unsteady thermal conduction, steady subsonic inviscid flow					18/11/2024, 20/11/2024, 22/11/2024	CO2

Textbook :	CFD by Anderson	#HOURS	Allotted	Taken
Signatures	Faculty: [signature] 23/11/2024		10	12
	HoD:			
Remarks	Module II Completed			

MODULE – 3

Lecture #	Topic	Mode of Delivery (PlsTick √) 1	2	3	4	Date of Delivery	COs Covered
21.	**Grid Generation and Adaptive Grids:** Need for grid generation and Body-fitted coordinate system.	✓				22/11/2021 23/11/2021	CO3
22.	Structured Grids-essential features	✓				24/11/2021	CO3
23.	Structured Grid generation techniques- algebraic and numerical methods.	✓				29/11/2021 30/11/2021	CO3
24.	Unstructured Grids-essential features	✓				1/12/2021	CO3
25.	Unstructured Grid generation techniques- Delaunay-Voronoi diagram,	✓				2/12/2021 3/12/2021	CO3
26.	Unstructured Grids- advancing front method.	✓				7/12/2021	CO3
27.	Surface grid generation, Multi-block grid generation	✓				8/12/2021	CO3
28.	Meshless methods. Grid quality and adaptive grids	✓				9/12/2021	CO3
29.	Structured grids adaptive methods and	✓				10/12, 11/12/2021	CO3
30.	Unstructured grids adaptive methods	✓				12/12/13/12, 14/12	CO3

Textbook : CFD by Anderson

Signatures		#HOURS	Allotted	Taken
	Faculty: [signature] 14/12/2021		10	16
	HoD:			

Remarks Module III Completed

MODULE – 4

Lecture #	Topic	Mode of Delivery (PlsTick √) 1	2	3	4	Date of Delivery	COs Covered
31.	**Discretization & Transformation** Discretization: Finite differences methods, and difference equations	✓				16/12/2022	CO4
32.	Explicit and Implicit approaches	✓				20/12/2022	CO4
33.	Unsteady Problems- Explicit versus Implicit Scheme.	✓				21/12/2022	CO4
34.	Errors and stability analysis.	✓				23/12/2022	CO4
35.	Time marching and space marching	✓				3/1/2022	CO4
36.	Reflection boundary condition. Relaxation techniques.	✓				4/1/2022	CO4
37.	Alternating direction implicit method	✓				10/1/2022	CO4
38.	Successive over relaxation/under Relaxation.	✓				11/1/2022	CO4
39.	Second order Lax-Wendorff method, mid-point Leap frog method,	✓				12/1/2022	CO4
40.	upwind scheme, numerical viscosity, and artificial viscosity	✓				12/1/2022	CO4
41.	**Transformation:** Transformation of governing partial differential equations from physical domain to computational domain.	✓				13/1/2022	CO4
42.	Matrices and Jacobians of transformation, Example of transformation.	✓				13/1/2022	CO4
43.	Generic form of the Governing flow equations in Strong Conservative form in the Transformed Space.	✓				13/1/2022	CO4

Textbook:

Signatures		#HOURS	Allotted	Taken
	Faculty: [signature] 17/1/2022			
	HoD:			

Remarks Module IV Completed

MODULE – 5							
Lecture #	Topic	Mode of Delivery (PlsTick √)				Date of Delivery	COs Covered
		1	2	3	4		
44.	**Module -5 Finite Volume Technique and Some Applications:** Spatial discretization		✓			17/1/2022	CO5
45.	cell centered and cell vertex techniques		✓			17/1/2022	CO5
46.	Overlapping control volume, duel control volume)		✓			17/1/2022	CO5
47.	Temporal discretization- Explicit time stepping, and implicit time stepping.		✓			17/1/2022	CO5
48.	Time step calculation		✓			18/1/2022	CO5
49.	Upwind scheme and high resolution scheme.		✓			18/1/2022	CO5
50.	Flux vector splitting, Approximate factorization		✓			18/1/2022	CO5
51.	Artificial dissipation and flux limiters		✓			18/1/2022	CO5
52.	Unsteady flows and heat conduction problems, Upwind biasing		✓			18/1/2022	CO5
Textbook :							
Signatures	Faculty:	#HOURS				Allotted	Taken
	HoD:						
Remarks							

Text Books:

1.Fletcher, C.A.J., "Computational Techniques for Fluid Dynamics". 2nd edition. Springer. Berlin. 2002.ISBN-13: 978-3540543046

2.John D. Anderson, "Computational Fluid Dynamics" McGraw Hill, 2013. **ISBN-13:** 978-0070016859.

Reference Books:

1. John F. Wendt, "Computational Fluid Dynamics - An Introduction".3rd edition, Springer, 2013
2. Charles Hirsch, "Numerical Computation of Internal and External Flows" 1st edition, Elsevier, 2007, ISBN-13: 978-9381269428.
3. Klaus A Hoffmann and Steve T. Chiang. "Computational Fluid Dynamics for Engineers", Vols. I & II Engineering Education System67208 - 1078 USA,1993.

(Note: Mode of Delivery : 1 Black Board 2 PPT 3 Video 4 Demo/Hands-on)

INTERNAL/ASSIGNMENT/QUIZ SCHEDULE

TEST and QUIZ		COs and Portions Covered		ASSIGNMENT	
Test# and Quiz#	DATE	CO	Modules	Assignment#	DATE
T1 & Q1	23/11/2021	CO1, CO2	I, II	A1	22/11/2021
T2 & Q2	23/12/2021	CO3, CO4	II, III, IV	A2	22/12/2021
T3 & Q3	19/1/2022	CO4, CO5	IV, V	A3	Mini Project.

SUMMARY

Signatures With Date	Faculty:	Total #HOURS	Allotted	Taken
	HoD:			
Remarks				

ENCLOSURES

1. Syllabus
2. CO Attainment
3. Gap Analysis
4. Special lectures/talks arranged if any

Feedback by PAC

Faculty Course coordinator PAC HOD

||JAI SRI GURUDEV||

S.J.C. INSTITUTE OF TECHNOLOGY

DEPARTMENT OF AERONAUTICAL ENGG.

COMPUTATIONAL FLUID DYNAMICS - 18AE72

QUESTION BANK - I

Sl. No.	Questions	CO	Levels
	MODULE I		
1.	Explain the different computer architectures used in CFD	CO1	L2
2.	Explain the importance of CFD in modern study and the solution of problems in Fluid Mechanics	CO1	L2
3.	With the help of a neat sketch explain the method of solving flow field over a supersonic blunt-nosed body	CO1	L2
4.	Describe the CFD ideas to understand and its application	CO1	L2
5.	With the help of neat sketches explain the different models of the flow.	CO1	L2
6.	Derive an expression for Substantial Derivative with usual notations	CO1	L3
7.	Derive an expression for Time Rate of change following a moving fluid element	CO1	L3
8.	Explain the models of the flows to obtain Conservation and Non-conservation form of Equations	CO1	L2
9.	Derive an expression for Divergence of Velocity with usual notations	CO1	L3
10.	With the help of a neat sketch explain the models of Finite Control Volume.	CO1	L2
11.	With the help of a neat sketch explain the models of Infinitesimal Fluid Element	CO1	L2
12.	Explain the physical significance of Divergence of Velocity and derive an expression for the same with usual notations.	CO1	L3
13.	Derive Continuity Equation considering a model of the finite control volume fixed in space with usual notations	CO1	L3
14.	Explain the method Shock fitting in CFD	CO1	L2
15.	Derive Continuity Equation considering a model of the finite control volume moving with the fluid with usual notations	CO1	L3
16.	Explain the method of Shock Capturing in CFD	CO1	L2
17.	Derive Continuity Equation considering a model of infinitesimally small element fixed in space with usual notations	CO1	L3
18.	Explain the differences between Integral and Differential forms of equations	CO1	L2
19.	Derive Continuity Equation considering a model of infinitesimally small element moving with the flow with usual notations	CO1	L3
20.	Explain the physical boundary conditions used in CFD	CO1	L2

21.	Derive the Momentum Equation considering an infinitesimally small fluid element moving with the flow, for unsteady, three-dimensional, compressible and viscous flow with usual notations	CO1	L3
22.	List down the comments done on the different types of governing equations and explain.	CO1	L3
23.	Derive the Energy Equation considering an infinitesimally small fluid element moving with the flow, with usual notations	CO1	L3
24.	Explain the different forms of the governing equations particularly suited for CFD work.	CO1	L2
	MODULE II		
25.	Explain the classification of Quasi-Linear Partial Differential Equations	CO2	L2
26.	Apply Eigen Value method to a Quasi-linear partial differential equation for the mathematical classification as elliptic, parabolic and hyperbolic.	CO2	L3
27.	Apply Cramer Rule to a Quasi-linear partial differential equation for the mathematical classification as elliptic, parabolic and hyperbolic.	CO2	L3
28.	Explain the different mathematical behavior of CFD equations that reflects different physical behavior of flow; give an example in each case.	CO2	L2
29.	Assuming a system of quasi-linear equations, with the help of a characteristic curve at a point p(x,y), classify the different types of PDE's stating example for each type.	CO2	L3
30.	Determine the type of PDE for the following equation: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ $\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0$ Also if a variable Φ is introduced such that $u = \frac{\partial \phi}{\partial x}$ and $v = \frac{\partial \phi}{\partial y}$ Determine the nature of PDE.	CO2	L4
31.	Describe the essential features of hyperbolic equation impact on physical behavior of CFD problems.	CO2	L3
32.	Explain the following with relevant sketches i) Steady Inviscid Supersonic Flow ii) Unsteady Inviscid Flow	CO2	L2
33.	Describe the essential features of parabolic equation impact on physical behavior of CFD problems.	CO2	L3
34.	Explain the following with relevant sketches i) Steady Boundary Layer Flows ii) Parabolized Viscous Flow iii) Unsteady Thermal Conduction	CO2	L3
35.	For the one dimensional unsteady thermal conduction through a semi-infinite fluid, write the governing equation, boundary conditions and plot the typical solution characteristics.	CO2	L3
36.	Consider the irrotational, 2-D steady flow of a compressible gas. The flow field is slightly perturbed from free stream like flow over a thin profile. Find the roots of equations involved in such kind of flow problem, using Eigen method.	CO2	L4

PROFESSOR & HEAD
Department of Aeronautical Engineering
S.J.C. Institute of Technology
CHICKBALLAPUR-562101

SJCIT

Name of the staff: Mrs. Deepa M S Date: 23.11.2021	Signature: Kefame 23/11/2021
Reviewer's Signature:	B... 23/11/2021

S J C Institute of Technology

Department of Aeronautical Engineering

Continuous Internal Evaluation: I

Semester: VII

Subject Name & Code: COMPUTATIONAL FLUID DYNAMICS – 18AE72

Duration: 90 minutes
Date: 25.11.2021

Max Marks: 50+10(MCQs)
Time: 2:00 to 3:30 PM

Question Number		Marks	CO	Bloom's Level
	PART A			
1	a) Explain the different computer architectures used in CFD	5	CO1	L2
	b) Describe the method of Shock Capturing in CFD	5	CO1	L2
	OR			
2	a) With the help of neat sketches explain the different models of the flow.	5	CO1	L2
	b) Describe the physical boundary conditions used in CFD	5	CO1	L2
3	Develop Continuity Equation considering a model of the finite control volume fixed in space with usual notations	10	CO1	L3
	OR			
4	Considering an infinitesimally small fluid element moving with the flow, for unsteady, three-dimensional, compressible and viscous flow with usual notations arrive at the Momentum Equation	10	CO1	L3
5	Derive an expression for Substantial Derivative with usual notations	10	CO1	L3
	OR			
6	Explain the physical meaning for Divergence of Velocity and derive an expression with usual notations.	10	CO1	L3
7.	Apply Cramer's Rule to a Quasi-linear partial differential equation for the mathematical classification as elliptic, parabolic and hyperbolic.	10	CO2	L3
	OR			
8.	Describe the essential features of hyperbolic equation impact on physical behavior of CFD problems.	10	CO2	L3
9	Illustrate the following with relevant sketches i) Steady Boundary Layer Flows ii) Parabolized Viscous Flow	10	CO2	L2
	OR			
10	Explain the following with relevant sketches i) Steady Inviscid Supersonic Flow ii) Unsteady Inviscid Flow	10	CO2	L2
	PART B - MULTIPLE CHOICE QUESTIONS			
1.	CFD is the third approach for fluid flow analysis. What are the other two approaches? a) Theoretical and experimental b) Physical and Mathematical c) Numerical and experimental d) Experimental and physical	1	CO1	L1

2.	CFD carries out ___________ experiments. a) Observational b) Analytical c) Field d) Numerical	1	CO1	L1
3.	This created a problem in modelling supersonic blunt nose. a) Change of flow equations from elliptic to hyperbolic b) High speed with high temperature c) Supersonic Mach number d) High temperatures	1	CO1	L1
4.	What is the advantage of numerical methods over analytical method? a) Speed b) Cost c) Flexibility d) Time	1	CO1	L1
5.	Which of these plots are irrelevant to CFD post-processing? a) Contour plots b) Vector plots c) xy plots d) Bar plots	1	CO1	L1
6.	An equation modelled using infinitesimally small element leads to a) Partial differential equation b) Integral equation c) Differential equation d) Linear differential equation	1	CO2	L1
7.	What are the two major types of boundary conditions? a) Wall and symmetry b) Inlet and outlet c) Dirichlet and Neumann d) Initial and physical	1	CO2	L1
8.	Which of these is not a type of flows based on their mathematical behavior? a) Circular b) Elliptic c) Parabolic d) Hyperbolic	1	CO2	L1
9.	The lines along which the derivatives of the dependent variables are indeterminate are called ___________ a) parabolic lines b) characteristic lines c) hyperbolic lines d) transition lines	1	CO2	L1
10	What are the two methods used to find the type of PDEs? a) Lagrangian Method and Eulerian method b) Cramer's method and Eulerian method c) Cramer's method and Lagrangian Method d) Cramer's method and Eigenvalue method	1	CO2	L1

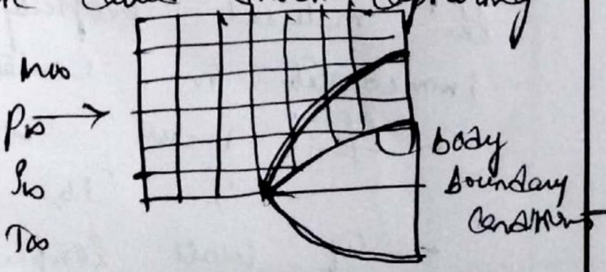
COURSE OUTCOMES: On successful completion of this course, students should be able to

CO-1	Describe the basics of CFD and parallel computing and explain the various flow models, its governing equations of fluid motion in differential and integral form. (L3)
CO-2	Compare the physics of the flow with the mathematical behavior of partial differential equations. (L3)
CO-3	Identify and compute the suitable grid generation and transformation techniques for a given problem. (L4)
CO-4	Illustrate the fundamentals of discretization in FVM, FDM and FEM techniques. (L3)
CO-5	Interpret different schemes and their stability in simple CFD applications. (L4)

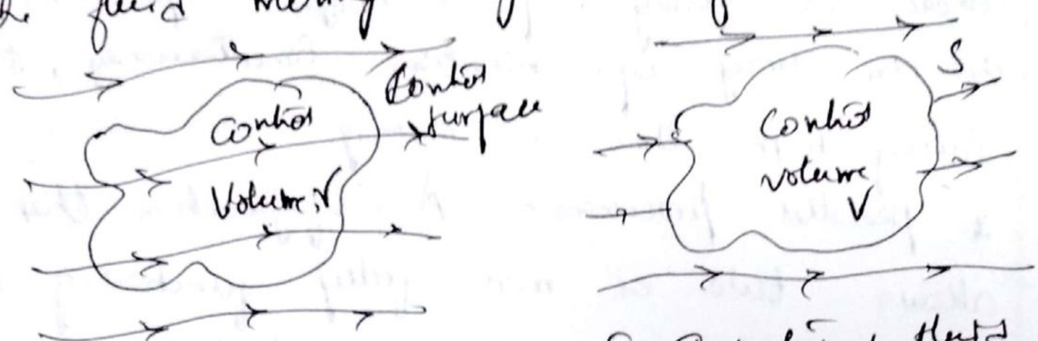
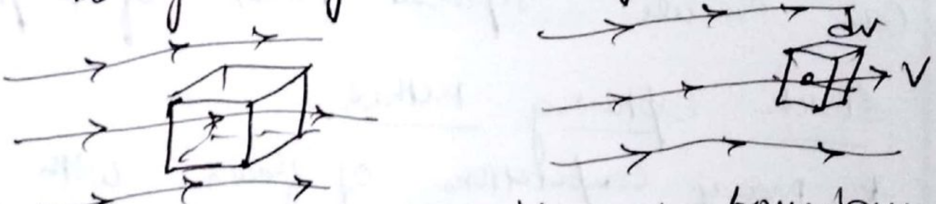
Department - Aeronautical Engineering

TEST I

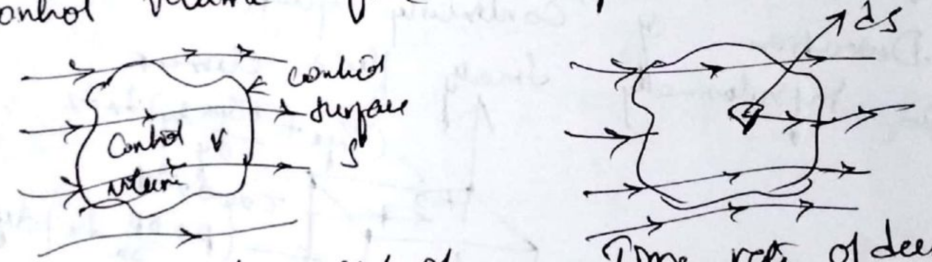
Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE72

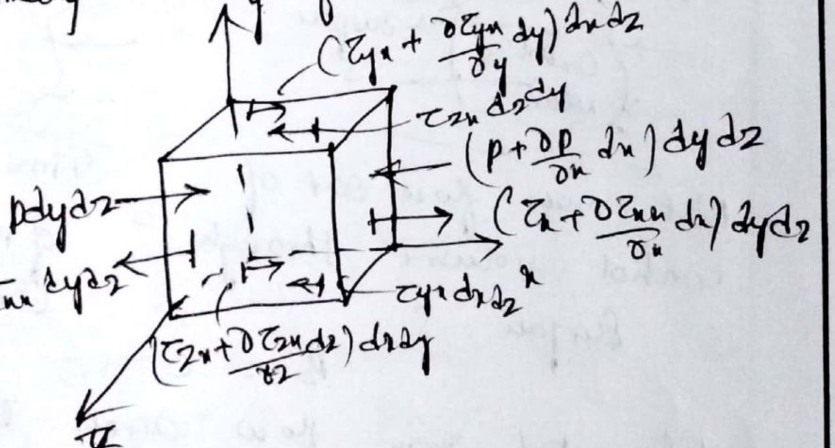
Question Number	Solution	Marks Allocated
1 a)	* Two Computer Architectures used in CFD - Vector processors - parallel processors	-1-
	* Vector processors - A Configuration that allows a string of identical operations on an array of numbers simultaneously, thus saving both time & memory.	-2-
	* parallel processors - A Configuration that allows two or more fully functioning central processing units each of which can handle different instructions and data streams and can execute separate parts of a program	-2-
		5
b)	Shock Capturing method	
	* Many Computations of flows with shocks are designed to have the shock waves appear naturally within the computational space as a direct result of the overall flow field solution ie as a direct result of the general algorithm, without any special treatment to take care of the shocks themselves. Such approaches are called 'shock capturing method'.	-3-
		-2-
		5

Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE72

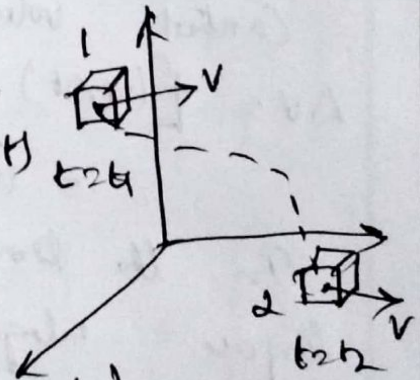
Question Number	Solution	Marks Allocated
2	a) Explanation of the four types of flow models	-1-
	I Finite control volume fixed in space with the fluid moving through it; II Finite control volume moving with the fluid	
		-2-
	III Infinitesimal fluid element fixed in space with the fluid moving through it; IV Infinitesimal fluid element moving along a streamline	
		-2-
		5
	b) * Explanation of the physical boundary conditions such as	-1-
	- If the surface is stationary with the flow moving past it, then at the surface (for viscous flow) $u=v=w=0$	
	- 'no slip' condition associated with the temperature at the surface. If T_w is the material property, fluid layer temperature immediately in contact with the surface is T_w.	-2-
	- For known wall temperature $T = T_w$ (at the wall)	
	- If wall temperature is unknown	

Subject Title: Computational Fluid Dynamics **Subject Code:** 18 AE 72

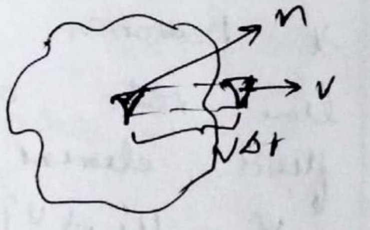
Question Number	Solution	Marks Allocated
	$\dot{q}_w = -k\left(\frac{\partial T}{\partial n}\right)_w$ (at the wall)	
	* Unsteady heat transfer problem the wall temperature itself is the boundary condition	
	$\left(\frac{\partial T}{\partial n}\right)_w = -\frac{\dot{q}_w}{k}$ (at the wall)	-2-
	- adiabatic wall $\left(\frac{\partial T}{\partial n}\right)_w = 0$ (at the wall)	
	* Fixed wall temperature is the easiest to apply and then the adiabatic wall	5
3.	* Derivation of continuity equation for finite control volume fixed in space	
		-2-
	Net mass flow out of control volume through surface S = Time rate of decrease of mass inside control volume	-2-
	$B = C$	
	Elemental mass flow across the area dS	
	$\rho V_n dS = \rho v \, dS$	
	$\therefore B = \iint_S \rho v \, dS$	-2-
	* Total mass inside the control volume is $\iiint_V \rho \, dv$	
	Time rate of increase of mass inside V is	
	$\frac{D}{\partial t} \iiint_V \rho \, dv$	-2-

Question Number	Solution	Marks Allocated
	Time rate of decrease of mass	
	$-\frac{\partial}{\partial t}\iiint_V \rho\, dv = c$	
	$\therefore \iint_S \rho v\, ds = -\frac{\partial}{\partial t}\iiint_V \rho v.$	-2-
	$\therefore \frac{\partial}{\partial t}\iiint_V \rho\, dv + \iint_S \rho v\, ds = 0.$	
	The above equation is the Integral form of continuity equation and is called conservation form of governing equation	10
4.	Derivation of momentum equation considering an infinitesimally small fluid element.	
		-2-
	$f_x = ma_x$ forces acting on the fluid element along x axis are Body forces.	-2-
	Surface forces - pressure distribution & shear normal stress distribution	
	Body forces on fluid element $= \rho f_x\,(dxdydz)$	
	Net surface forces on the moving fluid element in x direction	

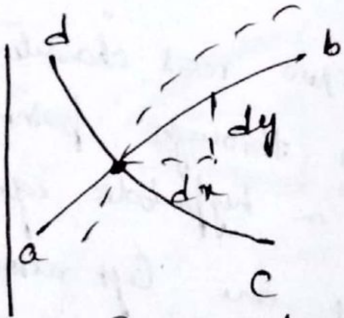
Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE72

Question Number	Solution	Marks Allocated
	$= \left[p - \left(p + \frac{\partial p}{\partial x} dx\right)\right] dy\,dz + \left[\left(\tau_{xx} + \frac{\partial \tau_{xx}}{\partial x} dx\right) - \tau_{xx}\right] dy\,dz + \left[\left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} dy\right) - \tau_{yx}\right] dx\,dz + \left[\left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} dy\right) - \tau_{zx}\right] dx\,dy$	-2-
	$f_x = \left[\frac{-\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right] dx\,dy\,dz$	-2-
	$a_x = \frac{Du}{Dt}$ $\quad \frac{\rho Du}{Dt} = \frac{-\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \rho f_x$	
	|ly $\frac{\rho Dv}{Dt} = \frac{-\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + \rho f_y$	-2-
	$\frac{\rho Dw}{Dt} = \frac{-\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} + \rho f_z$	10
5.	* Derivation of substantial derivative i.e. time rate of change following a moving fluid element	
		
	$V = ui + vj + wk$ $u = u(x, y, z, t)$ $\quad v = v(x, y, z, t)$ $w = w(x, y, z, t)$ $\rho = \rho(x, y, z, t)$	-2-
	$\rho_1 = \rho(x, y, z, t_1)$ $\quad \rho_2 = \rho(x, y, z, t_2)$	
	$\rho_2 = \rho_1 + \left(\frac{\partial \rho}{\partial x}\right)_1 (x_2 - x_1) + \left(\frac{\partial \rho}{\partial y}\right)_1 (y_2 - y_1) + \left(\frac{\partial \rho}{\partial z}\right)_1 (z_2 - z_1)$	-2-

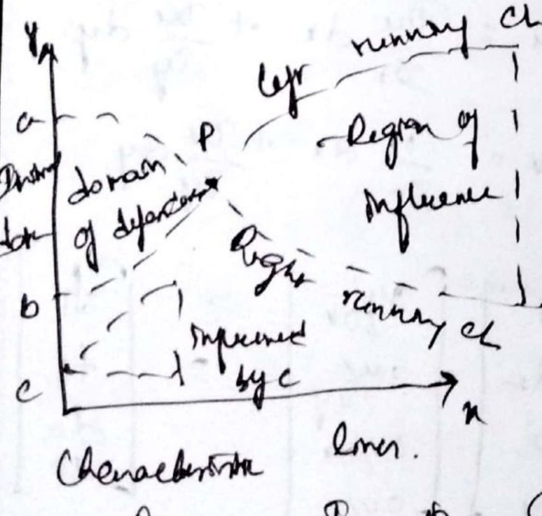
Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE

Question Number	Solution	Marks Allocated
	$\frac{\rho_2-\rho_1}{t_2-t_1} = \left(\frac{\partial \rho}{\partial x}\right)_1 \frac{x_2-x_1}{t_2-t_1} + \left(\frac{\partial \rho}{\partial y}\right)_1 \frac{y_2-y_1}{t_2-t_1} + \left(\frac{\partial \rho}{\partial z}\right)_1 \frac{z_2-z_1}{t_2-t_1} + \left(\frac{\partial \rho}{\partial t}\right)_1$	2
	$\lim_{t_2\to t_1} \frac{\rho_2-\rho_1}{t_2-t_1} = \frac{D\rho}{Dt}$; $\lim_{t_2\to t_1} \frac{x_2-x_1}{t_2-t_1} = u$; $\lim_{t_2\to t_1} \frac{y_2-y_1}{t_2-t_1} = v$; $\lim_{t_2\to t_1} \frac{z_2-z_1}{t_2-t_1} = w$.	2
	$\therefore \frac{D\rho}{Dt} = u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} + \frac{\partial \rho}{\partial t}$	
	$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$	2
	$\frac{D}{Dt} = \frac{\partial}{\partial t} + V.\nabla$	10
6.	Change in volume of the Control volume ΔV. 	2
	$\Delta V = [(V\Delta t).n]\, ds$. $= (V\Delta t)\, ds$	
	In the limit $ds \to 0$, the sum becomes surface integral $\iint_S (V\Delta t).\,ds$	
	$\therefore \frac{DV}{Dt} = \frac{1}{\Delta t}\iint_S (V.\Delta t)\, ds = \iint_S V\, ds$	2

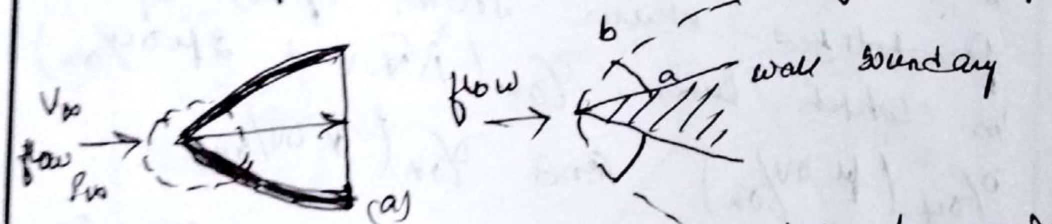
Subject Title: Computational fluid Dynamics **Subject Code:** 18AE72

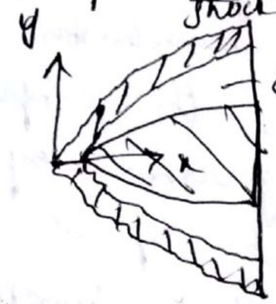
Question Number	Solution	Marks Allocated
	$\frac{DV}{Dt} = \iiint_V (\nabla \cdot V)\, ds$	-2-
	$\frac{D(\delta V)}{Dt} = \iiint_{\delta V} (\nabla \cdot V) dV.$	
	$\frac{D(\delta V)}{Dt} = (\nabla \cdot V)\, \delta V.$	
	$\nabla \cdot V = \frac{1}{\delta V} \frac{D(\delta V)}{Dt}$	-2-
	$\nabla \cdot V$ is physically the time rate of change of the volume of a moving fluid element per unit volume	-2-
		10
7.	A System of quasi linear equations	
	$a_1 \frac{\partial u}{\partial x} + b_1 \frac{\partial u}{\partial y} + c_1 \frac{\partial v}{\partial x} + d_1 \frac{\partial v}{\partial y} = f_1$ — 1	-2-
	$a_2 \frac{\partial u}{\partial x} + b_2 \frac{\partial u}{\partial y} + c_2 \frac{\partial v}{\partial x} + d_2 \frac{\partial v}{\partial y} = f_2$	
		
	$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy$	-2-
	$dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy$	
	$\begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ dx & dy & 0 & 0 \\ 0 & 0 & dx & dy \end{bmatrix} \begin{bmatrix} \partial u/\partial x \\ \partial u/\partial y \\ \partial v/\partial x \\ \partial v/\partial y \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ du \\ dv \end{bmatrix}$	-2-
	$[A] = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ dx & dy & 0 & 0 \\ 0 & 0 & dx & dy \end{bmatrix}$ $[B] = \begin{bmatrix} f_1 & b_1 & c_1 & d_1 \\ f_2 & b_2 & c_2 & d_2 \\ du & dy & 0 & 0 \\ dv & 0 & dx & dy \end{bmatrix}$	

Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE62

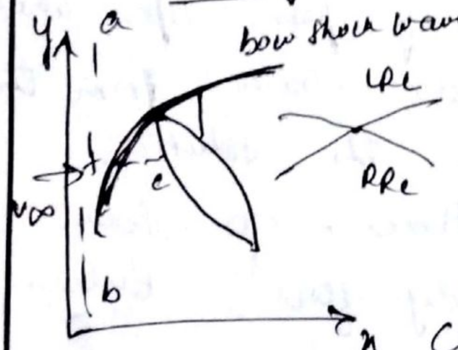
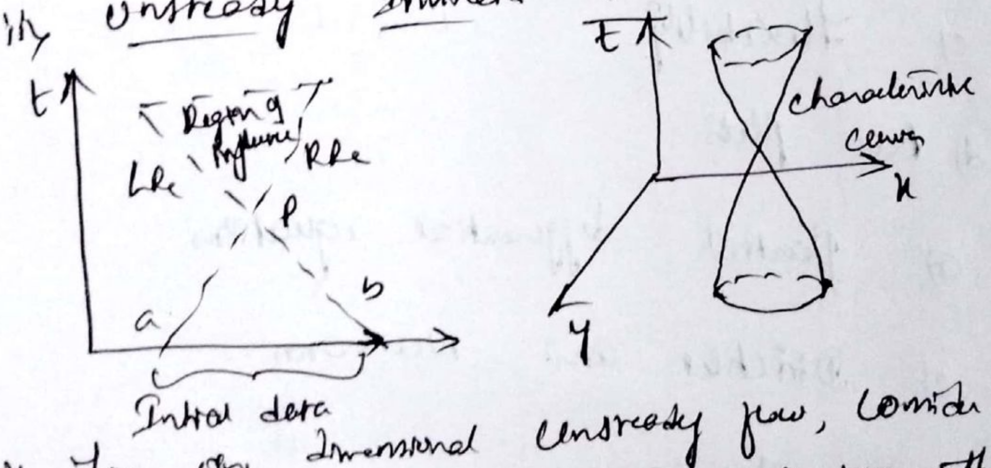
Question Number	Solution	Marks Allocated
	$\frac{\partial u}{\partial x} = \frac{\|B\|}{\|A\|}$ $\therefore \begin{vmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ dx & dy & 0 & 0 \\ 0 & 0 & dx & dy \end{vmatrix} = 0.$	
	$(a_1c_2 - a_2c_1)dy^2 - (a_1d_2 - a_2d_1 + b_1c_2 - b_2c_1)dx\,dy + (b_1d_2 - b_2d_1)dx^2 = 0.$	
	$a = a_1c_2 - a_2c_1$, $b = -(a_1d_2 - a_2d_1 + b_1c_2 - b_2c_1)$, $c = (b_1d_2 - b_2d_1)$	2
	$a(dy/dx)^2 + b(dy/dx) + c = 0.$	
	$\therefore dy/dx = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	
	$D = b^2 - 4ac$	
	If $D > 0$, Two real & distinct characteristics & hyperbolic	2
	$D = 0$, Parabolic	
	$D < 0$, Imaginary, elliptic	10
8.	 * Two real characteristic curves through point P for a hyperbolic equation. They are left running and right running characteristic lines.	2
		2
	* Region II is called the region of influence which influences only the region between the two characteristic lines and the disturbance is at every point.	2

Subject Title: Computational fluid Dynamics **Subject Code:** 18AE72

Question Number	Solution	Marks Allocated
	* Region III - that is information at point c which is outside the interval ab is propagates along characteristics c and influences only region II	-2-
	* Domain of dependence - region III and point P depends on only that part of the boundary which intercepted by and included between the two relieving characteristics	-2-
	* In the field of CFD, the computations of flow fields that are governed by hyperbolic equations is set up as marching solution. and the algorithm is designed to start with initial solutions	10
9	i) Steady boundary layer flow:	
		-2-
	* general flow is divided - into two regions - a thin layer adjacent to any solid surface where in all the viscous effects are contained and - an inviscid flow outside thin viscous	-1-
	* schematic of the boundary layer is shown in fig (a) and assumes that the boundary layer is thin and that Reynolds	

Question Number	Solution	Marks Allocated
	Re is based on body length L is large ($Re = \rho_\infty V_\infty L/\mu$) and the Navier Stokes equation is reduced to an approximate set of equations called boundary layer equations and are parabolic	-2- 5
	iii) parabolized viscous flow:-	
	 A supersonic flow over pointed nose body is as shown in fig. If the Reynolds number is low enough the viscous effects will reach well into the flow field. far away from the surface	-1-
	* If the flow is assumed to be steady then the resulting equations are called the parabolized navier stokes (PNS) equations in which terms $\partial/\partial x(\lambda \nabla . V + 2\mu \partial u/\partial x)$ $\partial/\partial y(\mu \partial v/\partial x)$ and $\partial/\partial x(k \partial T/\partial x)$	-2-
	* Advantages of PNS equations are - they are simpler - Can be solved by means of a downstream marching solutions	-2- 5

Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE72

Question Number	Solution	Marks Allocated
10.	i) Steady Inviscid Supersonic flow:	
		
	* If the flow is two-dimensional, the behaviour is hyperbolic. A supersonic flow over a two-dimensional circular-arc airfoil is as shown in fig. The airfoil can be at an angle of attack α, but α must not be so large as to cause the leading edge shock to become detached or else there will be pockets of subsonic flow in a steady flow.	-1- -2- -2-
	* Shock capturing or shock fitting method can be used to capture the shock.	5
	ii) Unsteady Inviscid flow:	
		-1-
	* For one dimensional unsteady flow, consider point P in the xt plane is as shown in fig. The region influenced by P is the shaded area between two advancing characteristic lines through P (t>0) is the initial data line	-2-

Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE72

Question Number	Solution	Marks Allocated
	& the region influenced by P and the portion of the boundary in the xy plane upon which the solution at P depends, starts from the known initial data and the solution marches forward in time. To solve three dimensional unsteady flow, Eulers equation are used.	-2
		5
	PART B	
1.	a) Theoritical and Experimental	-1
2.	d) Numerical	-1
3.	a) Change of flow equations from elliptic to hyperbolic	-1
4.	c) Flexibility	-1
5.	d) Bar plots	-1
6.	a) Partial differential equations	-1
7.	c) Dirichlet and Neumann	-1
8.	a) Circular	-1
9.	b) Characteristic lines	-1
10.	d) Cramers method and Eigen value method	-1
		10

Staff Incharge 11/12/2021 Reviewer 13/12/2021 HOD 13/12/2021

PROFESSOR & HEAD
Department of Aeronautical Engineering
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||JAI SRI GURUDEV||

S.J.C. INSTITUTE OF TECHNOLOGY

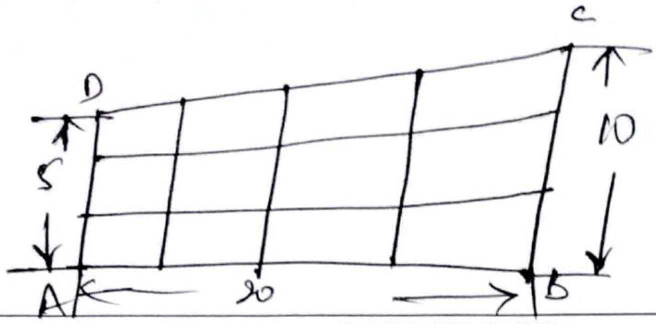
DEPARTMENT OF AERONAUTICAL ENGG.

FOR THE ACADEMIC YEAR 2021- 22

COMPUTATIONAL FLUID DYNAMICS - 15AE72

TUTORIALS II

Sl. No.	Questions	CO	Levels
	MODULE III		
1.	Define grid generation and describe the importance of boundary fitted co-ordinate system in CFD.	CO3	L2
2.	With the help of relevant sketches explain boundary fitted co-ordinate system	CO3	L2
3.	With the help of relevant sketches explain Elliptic Grid Generation	CO3	L2
4.	Explain the boundary fitted coordinate system for the divergent duct.	CO3	L2
5.	Describe elliptic grid generation with suitable example	CO3	L2
6.	List the features of Structured grids? Explain the different methods of structured grid generation with suitable sketches.	CO3	L2
7.	Explain the following with relevant sketches i) Algebraic Methods ii) Differential Equation Technique or Numerical Method	CO3	L2
8.	List the features of Unstructured grids? Explain the different methods of unstructured grid generation with suitable sketches.	CO3	L2
9.	Explain the following with relevant sketches i) Point Insertion Schemes or Delaunay-Voronoi Diagram ii) Advancing Front Methods	CO3	L2
10.	List the differences between structured and unstructured grids	CO3	L2
11.	Discuss the need of grid generation of grid generation in CFD and summarize the following methods of grid generation. i) Surface Grid Generation ii) Multi Block Grid Generation iii) Meshless Methods	CO3	L2
12.	Define Mesh/Grid Quality and explain in detail the different measures of grid quality	CO3	L2
13.	Discuss the following i) Striving for quality ii) Grid design guidelines and total cell count	CO3	L2
14.	With the help of relevant sketches explain the adaptive grid generation.	CO3	L2
15.	Differentiate Structured and Unstructured Adaptive grids with suitable sketches.	CO3	L2
16	List the essential properties of grids	CO3	L2
17	Explain the importance of various coordinate systems in grid generation	CO3	L2
18	Consider a Trapezoid in (x,y) plane as shown in fig. Generate a mesh corresponding to (ξ,η) coordinates at 0.2 units apart. Take	CO3	L3,L4

	four points of Trapezoid as A(0,0), B(20,0), C(20,10) and D(0,5) 		

MODULE IV

1.	Summarize the essence of discretization in CFD	CO4	L2
2.	Derive the Taylor Series approach for the construction of finite Difference Equation	CO4	L3
3.	With the help of neat grids explain the graphical concept of finite difference modules and list the pros and cons of higher order accuracy	CO4	L3
4.	Illustrate a portion of a boundary and explain what happens at the boundary. Also describe the reflection boundary conditions	CO4	L3
5.	Derive a difference equation for an unsteady one dimensional heat conduction equation with thermal diffusivity with usual notations.	CO4	L3
6.	Demonstrate the Explicit and Implicit approaches of solving CFD. List their advantages and disadvantages	CO4	L3

22/12/2021
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06#Form#02a - Rev. No. 02
Page: 1/3

Internal Test Question paper format – 2018 Scheme

Name of the staff: Mrs. Deepa M S
Date: 24.12.2021
Signature: Deepa 24/12/2021

Reviewer's Signature:	No J.D 24/12/2021

NOTE: Only the following information's to be given to the students.

S.J.C. Institute of Technology

Department of Aeronautical Engineering
Continuous Internal Evaluation: II
Semester: VII
Subject Name & Code: COMPUTATIONAL FLUID DYNAMICS – 18AE72

Instructions

Duration: 90 minutes
Date: 27.12.2021

Max Marks: 50+10(MCQs)
Time: 2:00 to 3:30 PM

Answer ALL THE QUESTIONS

Question Number	PART A	Marks	CO	Levels
1	a) Explain the boundary fitted coordinate system for the divergent duct.	5	CO3	L2
	b) List the features of structured grids? Describe the different methods of structured grid generation with suitable sketches.	5	CO3	L3
	OR			
2	a) Discuss the need of grid generation of grid generation in CFD and summarize the Surface Grid Generation	5	CO3	L2
	b) List the features of Unstructured grids? Explain the different methods of unstructured grid generation with suitable sketches.	5	CO3	L3
3	a) Define Mesh/Grid Quality and explain in detail the different measures of grid quality	5	CO3	L2
	b) Consider a Trapezoid in (x,y) plane as shown in fig. Generate a mesh corresponding to (ξ,η) coordinates at 0.2 units apart. Take four points of Trapezoid as A(0,0), B(20,0), C(20,10) and D(0,5)	5	CO3	L4
	OR			
4	a) List the essential properties of grids and explain	5	CO3	L2
	b) Consider a Trapezoid in (x,y) plane as shown in fig. Generate a mesh corresponding to (ξ,η) coordinates at 0.3 units apart. Take four points of Trapezoid as A(0,0), B(30,0), C(30,15) and D(0,5)	5	CO3	L4

5	Discuss the need of grid generation of grid generation in CFD and summarize the following methods of grid generation. i) Multi Block Grid Generation iii) Meshless Methods	10	CO4	L2
	OR			
6	Explain the following with relevant sketches i) Point Insertion Schemes or Delaunay-Voronoi Diagram ii) Advancing Front Methods	10	CO4	L2
7	With the help of neat grids explain the graphical concept of finite difference modules and list the pros and cons of higher order accuracy	10	CO4	L3
	OR			
8	Illustrate a portion of a boundary and explain what happens at the boundary. Also describe the reflection boundary conditions	10	CO4	L3
9	Derive the Taylor Series approach for the construction of finite Difference Equation	10	CO4	L3
	OR			
10	Derive a difference equation for an unsteady one dimensional heat conduction equation with thermal diffusivity with usual notations.	10	CO4	L3

PART B - MULTIPLE CHOICE QUESTIONS

.	When can we say that a problem is suitable to be solved using CFD? a) The PDE has no solution b) The solution to PDE is unique and it depends continuously on the initial and boundary conditions c) The PDE has more than one solution d) The solution to PDE is unique and independent of the initial and boundary conditions	1	CO3	L1
.	Discretization of the physical domain of interest results in ________ a) Boundaries b) Discretized equations c) Discrete cells d) Exponential equations	1	CO4	L1
.	Which of these methods is not a method of discretization? a) Finite volume method b) Finite difference method c) Spectral element method d) Finite element method	1	CO3	L1
.	Discretization of the governing equations result in ________ a) Integral equations b) Quasi-linear partial differential equations c) Partial differential equations d) Algebraic equations	1	CO4	L1
.	What is the main disadvantage of explicit schemes in a time-dependent problem? a) Marching solution b) Simultaneous equations c) Small time-step size d) Small grid size	1	CO4	L1

6.	What is advantageous in implicit schemes? a) Error b) Consistency c) Convergence d) Stability	1	CO4	L1
7.	Adaptive grids change automatically based on ______________ a) flow field gradients b) time rate of change of the flow properties c) grid gradients d) time rate of change of the grid points	1	CO3	L1
8.	Which type of grids is the best for flow over an airfoil? a) Stretched grids b) Adaptive grids c) Boundary-fitted grids d) Elliptic grids	1	CO4	L1
9.	If the domain and equations are not discretized, which of these will become true? a) Numerical solution cannot be obtained b) Analytical solution cannot be obtained c) Initial conditions cannot be applied d) Mathematical model cannot be obtained	1	CO3	L1
0	The discretized equation connects each element with __________ a) the northern and southern elements b) the boundary elements c) the neighbouring elements d) the eastern and western elements	1	CO4	L1

COURSE OUTCOMES:

On successful completion of this course, students should be able to

CO-1	Describe the basics of CFD and parallel computing and explain the various flow models, its governing equations of fluid motion in differential and integral form. (L3)
CO-2	Compare the physics of the flow with the mathematical behavior of partial differential equations. (L3)
CO-3	Identify and compute the suitable grid generation and transformation techniques for a given problem. (L4)
CO-4	Illustrate the fundamentals of discretization in FVM, FDM and FEM techniques. (L3)
CO-5	Interpret different schemes and their stability in simple CFD applications. (L4)

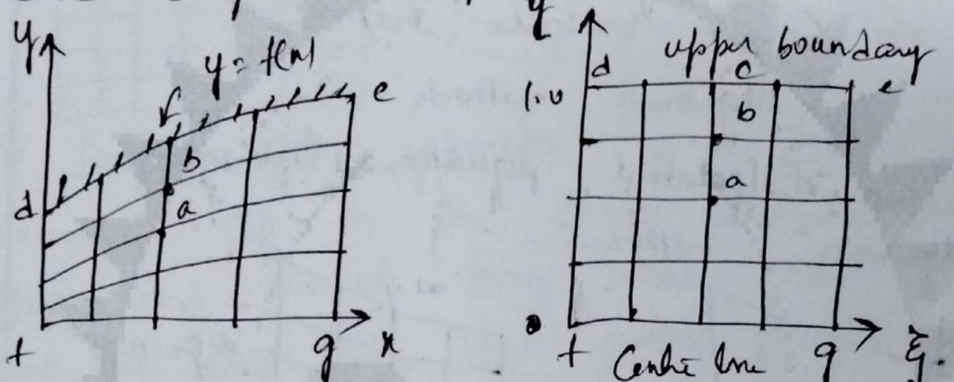
CIT
Page 1 of 10

06#Form#03 - Rev. No. 00

DEPARTMENT: AERONAUTICAL ENGINEERING

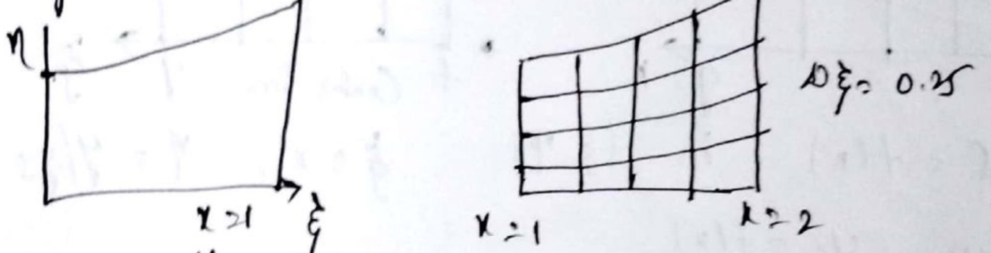
Scheme & Solutions- TEST- I/II/III **Date:** 24/12/2021

Semester: VII **Subject Title:** Computational Fluid Dynamics **Subject Code:** 18AE72

Question Number	Solution	Marks Allocated
1 a)	* Sketch of a simple boundary fitted co-ordinate system in physical plane and computational plane	
		-2-
	$y_s = f(x)$ in (ξ, η) $\xi = x$, $\eta = y/y_s$	-2-
	where $y_s = f(x)$ $\eta_d = y_d/y_s = \frac{y_s(x_d)}{y_s(x_d)} = 1$, $\eta_d = 1$	
	$\eta_c = \frac{y_c}{y_s} = \frac{y_s(x_c)}{y_s(x_c)} = 1.$	-1- 5
b)	* List of various features of structured grids - greatly simplifies the programming of the solution - saves storage space - results in greater accuracy	-1-
	* A given family of co-ordinate lines do not intersect the lines of constant ξ & do not cross lines of constant η	

Subject Title: Computational Fluid Dynamics **Subject Code:** 18A

Question Number	Solution	Marks Allocated
	Hence there is a structure to these grids such grids are called Structured grids	-1-
	* Structured grid generation consists of three categories - Complex variable methods - Algebraic methods - Differential equation techniques	-1-
	* Algebraic methods	-1-



$\xi = x$, $\eta = y/y_{max}$ $\quad \eta_x = -\dfrac{y}{y_{max}}\dfrac{dy_{max}}{dx} - \dfrac{2\eta}{\xi}$

$\eta_y = \dfrac{1}{y_{max}} = \dfrac{1}{\xi^2}$

* Numerical methods — -1-

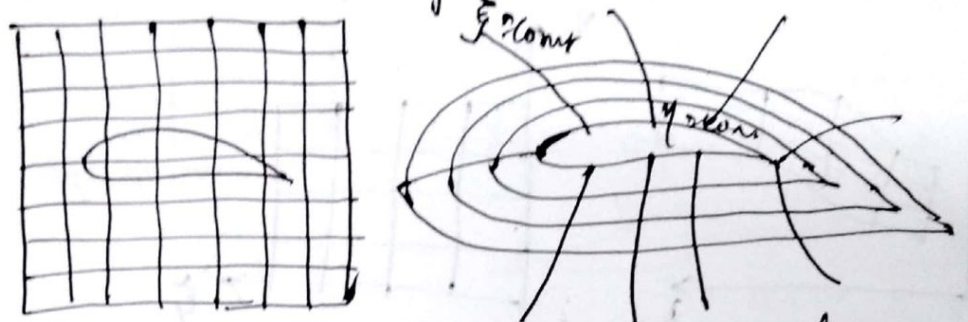
$\xi_x = \dfrac{y_\eta}{J}$ $\quad \xi_y = \dfrac{-x_\eta}{J}$ $\quad \eta_x = \dfrac{-y_\xi}{J}$

$\eta_y = \dfrac{x_\xi}{J}$ $\quad J = (J)^{-1} = x_\xi y_\eta - y_\xi x_\eta$ — 5

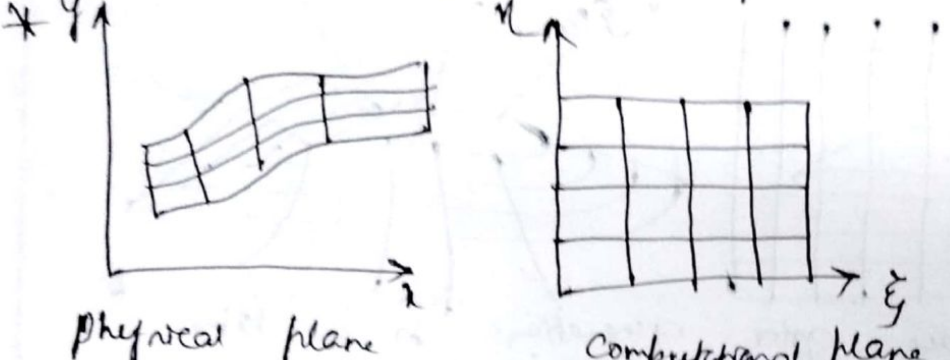
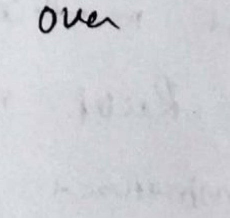
Question Number	Solution	Marks Allocated
2 a)	* The arrangement of discrete points throughout the flow field is called a grid. The way of determining these grids is called grid generation which involves the following	-1-

06#Form#03 - Rev. No. 00

Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE72

Question Number	Solution	Marks Allocated
	- The generation of an appropriate grid / mesh - The solution of the governing flow equation over such a grid	-1-
		-1-
	* Surface grid generation is a time consuming step in the overall process. Surface grids and models have value only as far as they allow high quality flow predictions to be made at an acceptable cost.	-1-
	* Very difficult to assess surface grid quality (orthogonality, curvature, stretching) Methods of constructing surface grid generation are Algebraic method, Laplace equation, Thomas and middlecoffin method	-1- 5
b).	* Recent research in CFD has focused on unstructured grids. When the grid points are placed in the flow field in a very irregular fashion. These non uniform grids in physical space can also be visualised as a mesh of finite volume cells.	-2-
	* Unstructured grids may be divided into three groups	

Subject Title: Computational **Subject Code:**

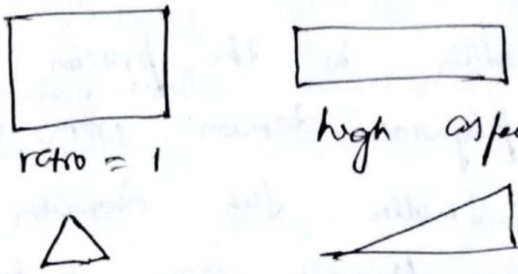
Question Number	Solution	Marks Allocated
	- point insertion Scheme - Advancing front methods - Domain dependence techniques	- 2 -
	* 	- 1 -
		5
3 a)	* For the same cell count, hexahedral meshes will give more accurate solutions, especially if the grid lines are aligned with the flow. The mesh density should be high enough to capture all relevant flow features	- 1 -
	* The mesh adjacent to the wall should be fine enough to resolve the boundary layer flow. In boundary layers, quad, hex and prism / wedge cells are preferred over tris, tets or pyramids.	- 1 -
	* Three measures of quality are - Skewness - Smoothness (Change in sizes - Aspect Ratio	- 1 -
	Skewness - $\frac{\text{Optimal cell size} - \text{Cell size}}{\text{Optimal cell size}}$ 	- 1

Subject Title: **Subject Code:**

Question Number	Solution	Marks Allocated

Skewness for quad = max $\left[\frac{\theta_{max}-90}{90}, \frac{90-\theta_{min}}{90}\right]$

Aspect Ratio —1—



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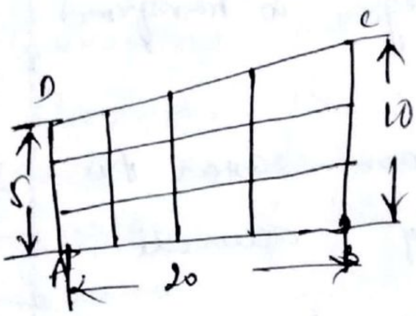
b)



$L = 20$, $H_1 = 5 = h$

$H_2 = 10 = H$ —1—

$y(x) = h + \frac{x(H-h)}{L}$

$\xi(x,y) = x/L$

$\eta(x,y) = y/Y = \frac{y}{h + \frac{x(H-h)}{L}}$ —1—

$y(x) = 5 + \frac{x(10-5)}{20} = 5 + \frac{5x}{20} = 5 + 0.25x$

$y(0.2) = 5 + 0.25 \times 0.2 = 5.05$ —1—

$\xi(x,y) = x/20$ $\eta(x,y) = \frac{y}{5+0.05x}$

$x(\xi,\eta) = \xi L$ —1—

$y(\xi,\eta) = \eta[h + \xi(H-h)]$ —1—

5

Subject Title: Computational fluid dynamics **Subject Code:** 18AE

Question Number	Solution	Marks Allocated
4 a)	List of essential properties of grids	-1-
	- poor quality grid will cause inaccurate solutions and/or slow convergence	-1-
	- Grid generation is the process of breaking up a physical domain with complex geometry into smaller sub domain.	-1-
	- A complex physical region is transformed to a simple computational region.	-1-
	- boundary conditions become easier to implement and approximately accurate.	-1-
		5
b)	A(0,0), B(30,0) C(30,15) D(0,5)	
	$L=30,\ H_1=5=h,\ H_2=15,H.$	-1-
	$y(x)=h+\frac{x(H-h)}{L}$ $\xi(x,y)=x/L$	
	$\eta(x,y)=y/Y=\frac{y}{h+\frac{x(H-h)}{L}}$	-1-
	$y(x)=5+\frac{x(15-5)}{30}=5+\frac{10x}{30}=5+0.3x$	-1-
	$y(0.3)=5+0.3\times0.3$	
	$=5+0.09$	-1-
	$y(0.3)=5.09$ $\xi(x,y)=x/30.$	+
	$\eta(x,y)=\frac{y}{5+0.3x}$	5

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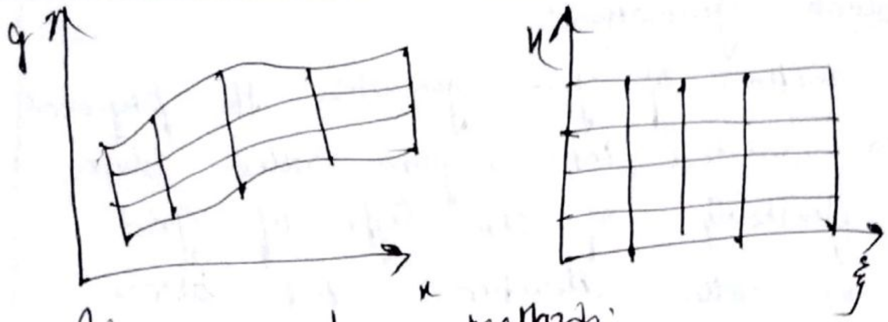
Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE72

Question Number	Solution	Marks Allocated
5 a)	i) Multi block generation: * In this method of grid generation the physical domain is divided into regions called blocks. Geometrical flexibility of this type of grid generation is better structured but worse than unstructured grids	- 2½
	* They are globally (in-between the blocks) unstructured but locally (within a block) structured. This block structuring can be viewed as compromise between high geometrical flexibility of fully unstructured grids and highly numerical efficient structured grids	- 2
	ii) Meshless methods: * Many mesh/grid generation procedures often lack automation, requiring many man hours which are becoming far more expensive than computer hardware	- 2
	Meshfree or meshless methods use a set of nodes scattered in a predefined manner within the problem domain No mesh implies no information on the relationship between the nodes is required	- 2½
		10
6. b)	i) point Insertion Schemes of Delaunay Voronoi diagram - Finite difference methods were restricted to problems where suitable coordinate systems could be selected in order to solve the governing equations in that base system	- 2½

Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE72

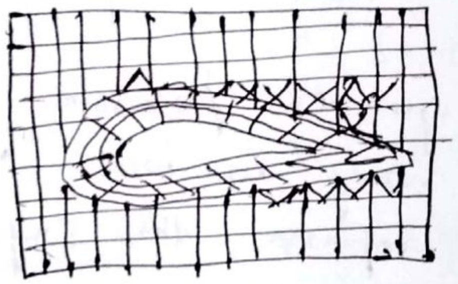
Question Number	Solution	Marks Allocated



– 2 –

(ii) Advancing front method:

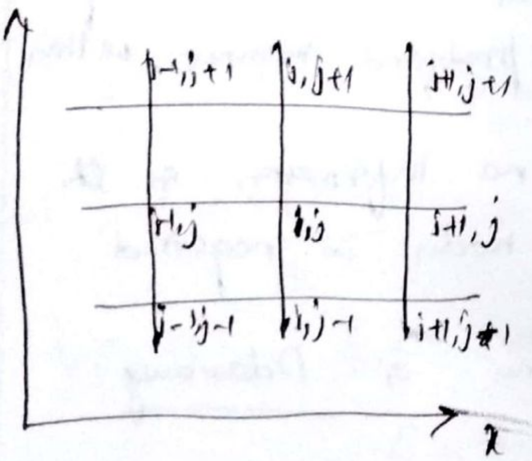
- These schemes of unstructured grid generation have been used with good success in a variety of application. With this approach the grid is advanced by adding cells at the front as it advances into the domain – 2½



– 2 –

10

7.

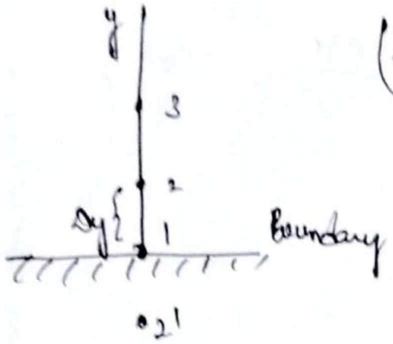
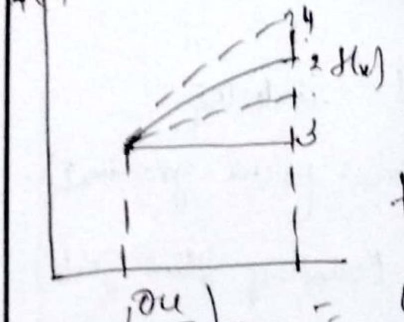


$$\left(\frac{\partial u}{\partial x}\right)_{i,j} = \frac{u_{i+1,j} - u_{i,j}}{\Delta x}$$ – first order forward difference – 2 –

$$\left(\frac{\partial u}{\partial x}\right)_{i,j} = \frac{u_{i,j} - u_{i-1,j}}{\Delta x}$$ – first order backward difference – 2 –

$$\left(\frac{\partial u}{\partial x}\right)_{i,j} = \frac{u_{i+1,j} - u_{i-1,j}}{2\Delta x}$$ – Second order Central difference – 2 –

Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE72

Question Number	Solution	Marks Allocated
	Der. of prom & con of finite difference	4
		10
8.		
	$\left(\frac{\partial u}{\partial y}\right)_1 = \frac{u_2 - u_1}{\Delta y} + O\,\Delta y$	
	$u_2' = u_2$	2
	$u = a + by + cy^2$ — Polynomial approach	
	$u_1 = a$, $u_2 = a + b\Delta y + c(\Delta y)^2$ at $\Delta y = y$	2
	$u_3 = a + b(2\Delta y) + c(2\Delta y)^2$ at $y = 2\Delta y$	
	$b = \frac{-3u_1 + 4u_2 - u_3}{2\Delta y}$ $\frac{\partial u}{\partial y}' = b + 2cy$	2
	$\left(\frac{\partial u}{\partial y}\right)_1 = b$ $\left(\frac{\partial u}{\partial y}\right)_1 = \frac{-3u_1 + 4u_2 - u_3}{2\Delta y}$	
	$u(y) = u_1 + \left(\frac{\partial u}{\partial y}\right)_1 y + \left(\frac{\partial^2 u}{\partial y^2}\right)_1 \frac{y^2}{2} + \left(\frac{\partial^3 u}{\partial x^3}\right)_1 \frac{y^3}{6} + \cdots$	2
	$\left(\frac{\partial u}{\partial y}\right)_1 = \frac{-3u_1 + 4u_2 - u_3}{2\Delta y} + O\,\Delta y^2$	2
		10
9.	$u_{i+1,j} = u_{i,j} + \left(\frac{\partial u}{\partial x}\right)_{i,j} \Delta x + \left(\frac{\partial^2 u}{\partial x^2}\right) \frac{\Delta x^2}{2} +$ $\left(\frac{\partial^3 u}{\partial x^3}\right)_{i,j} \frac{\Delta x^3}{6} + \cdots$	2
		
	$f(x+\Delta x) = f(x) + \frac{\partial f}{\partial x}\Delta x + \frac{\partial^2 f}{\partial x^2}\frac{(\Delta x)^2}{2} + \cdots \frac{\partial^n f}{\partial x^n}\frac{\Delta x^n}{n!} + \cdots$	2
	$f(x) + \frac{\partial f}{\partial x}\Delta x + \frac{\partial^2 f}{\partial x^2}\frac{\Delta x^2}{2} + \cdots$	
	$\left(\frac{\partial u}{\partial x}\right)_{i,j} = \frac{u_{i+1,j} - u_{i,j}}{\Delta x} - \left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j}\frac{\Delta x}{2} - \left(\frac{\partial^3 u}{\partial x^3}\right)_{i,j}\frac{\Delta x^2}{6} + \cdots$	2
		2

Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE72

Question Number	Solution	Marks Allocated

$$\left(\frac{\partial u}{\partial x}\right)_{i,j} = \frac{u_{i+1,j} - u_{i,j}}{\Delta x} + O\Delta x$$

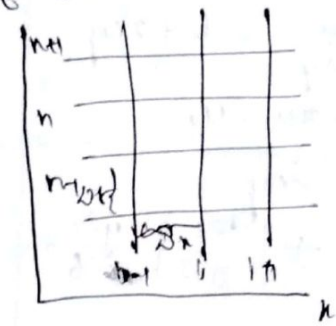
$$u_{i-1,j} = u_{i,j} + \left(\frac{\partial u}{\partial x}\right)_{i,j}(-\Delta x) + \left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j}\frac{-\Delta x^2}{2} + \left(\frac{\partial^3 u}{\partial x^3}\right)_{i,j}\frac{-\Delta x^3}{6} + \cdots$$ — 2

— 20

10.

$$\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial x^2}$$ — 2

$$\left(\frac{\partial T}{\partial t}\right)_i^n = \frac{T_i^{n+1} - T_i^n}{\Delta t} - \left(\frac{\partial^2 T}{\partial t^2}\right)\frac{\Delta t}{2} + \cdots$$



$$\left(\frac{\partial^2 T}{\partial x^2}\right)_i^n = \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{(\Delta x)^2} - \frac{\partial^4 T}{\partial x^4}\frac{(\Delta x^2)}{12} + \cdots$$ — 2

$$\frac{\partial T}{\partial t} - \alpha\frac{\partial^2 T}{\partial x^2} = 0$$ — 2

$$\frac{\partial T}{\partial t} - \alpha\frac{\partial^2 T}{\partial x^2} = 0 \quad \frac{T_i^{n+1} - T_i^n}{\Delta t} - \alpha\frac{(T_{i+1}^n - 2T_i^n + T_{i-1}^n)}{\Delta x^2}$$ — 2

$$-\left(\frac{\partial^2 T}{\partial t^2}\right)^n\frac{\Delta t}{2} + \alpha\left(\frac{\partial^4 T}{\partial x^4}\right)_i^n\frac{\Delta x^2}{12} + \cdots$$] Difference equation — 2

Truncation error

— 10

Part B

1.	b - depends on initial & boundary conditions	6. d stability
2.	c - Discrete cells	7. a flow field gradients
3.	c - Spectral element method	8. c - Boundary fitted grids
4.	d - Algebraic equation	9. a. Numerical form computer data
5.	c - Small time step δt	10. c. To neighbouring element

1×10 = 10.

Staff Incharge 24/12/2021   Reviewer 24/12/21   HOD.

PROFESSOR & HEAD
Department of Aeronautical Engineering
S.J.C. Institute of Technology

||JAI SRI GURUDEV||

S.J.C. INSTITUTE OF TECHNOLOGY

DEPARTMENT OF AERONAUTICAL ENGG.

FOR THE ACADEMIC YEAR 2021- 22

COMPUTATIONAL FLUID DYNAMICS - 15AE72

TUTORIALS III

Sl. No.	Questions	CO	Levels
	MODULE IV		
1.	Summarize the essence of discretization in CFD	CO4	L2
2.	Derive the Taylor Series approach for the construction of finite Difference Equation	CO4	L3
3.	With the help of neat grids explain the graphical concept of finite difference modules and list the pros and cons of higher order accuracy	CO4	L3
4.	Derive a difference equation for an unsteady one dimensional heat conduction equation with thermal diffusivity with usual notations.	CO4	L3
5.	Demonstrate the Explicit and Implicit approaches of solving CFD. List their advantages and disadvantages	CO4	L3
6.	Explain the different types of errors involved in solving finite difference equations and summarize the stability analysis	CO4	L2
7.	Explain Time marching technique with relevant equations	CO4	L2
8.	What is numerical dissipation and dispersion	CO4	L2
9.	With the help of neat grids and equations explain i) Relaxation Technique ii) Alternating Direct Implicit (ADI) Method iii) Successive Over relaxation and Under Relaxation iv) Second Order Lax-Wendorff Method v) Mid-Point Leap Frog Method vi) Upwind Scheme	CO4	L3
10.	Derive the expression for amplification factor and stability requirement for a numerical solution using Lax-Wendroff technique using 1D wave equation with one-step scheme.	CO4	L3
11.	Derive the expression for amplification factor and stability requirement for a numerical solution using central differencing for 1D unsteady heat conduction equation with genera notations..	CO4	L3
	MODULE V		
13.	Describe the finite volume discretization method.	CO4	L2
14.	Differentiate between the finite difference method and finite volume method.	CO4	L2
15.	Derive finite volumeupdate formula for 1D linear convective equation using upwind scheme.	CO4	L3
16	Derive finite volume update formula for 1D unsteady heat conduction equation using centered scheme.	CO4	L3
17	Explain upwind scheme in finite volume method.	CO4	L2
18	Explain Flux vector splitting method.	CO4	L2
19	Explain numerical viscosity, artificial dissipation and its mitigation method in finite volume discretization.	CO4	L2
20	Differentiate between the cell center finite volume approach and cell vertex finite volume approach.	CO4	L2

17/1/2022

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06#Form#02a - Rev. No. 02
Page: 1/2

Internal Test Question paper format – 2018 Scheme

Name of the staff: Mrs. Deepa M S Date: 17.01.2022	Signature: Deepa 17/1/2022

Reviewer's Signature:	B.v 17/1/2022

NOTE: Only the following information's to be given to the students.

S.J.C. Institute of Technology

Department of Aeronautical Engineering
Continuous Internal Evaluation: III
Semester: VII
Subject Name & Code: COMPUTATIONAL FLUID DYNAMICS – 18AE72

Instructions

Duration: 90 minutes
Date: 19.01.2022

Max Marks: 50
Time: 2:00 to 3:30 PM

Answer ALL THE QUESTIONS

Question Number		Marks	CO	Bloom's Level
PART A				
1	Explain the different types of errors involved in solving finite difference equations and summarize the stability analysis	10	CO4	L2
OR				
2	With the help of neat grids and equations explain Relaxation Technique	10	CO4	L2
3	Derive a difference equation for an unsteady one-dimensional heat conduction equation with thermal diffusivity with usual notations.	10	CO4	L3
OR				
4	Derive the expression for amplification factor and stability requirement for a numerical solution using Lax-Wendroff technique using 1D wave equation with one-step scheme.	10	CO4	L3
5	With the help of neat grids and equations explain Mid-Point Leap Frog Method	10	CO4	L3
OR				
6	With the help of neat grids and equations explain Alternating Direct Implicit (ADI) Method	10	CO4	L3
7.	Differentiate between the cell center finite volume approach and cell vertex finite volume approach.	10	CO5	L2
OR				

8.	Differentiate between the finite difference method and finite volume method.	10	CO5	L2
9	Describe numerical viscosity, artificial dissipation and its mitigation method in finite volume discretization.	10	CO5	L2
	OR			
10	Illustrate Flux vector splitting method with usual notations	10	CO5	L2

COURSE OUTCOMES:

On successful completion of this course, students should be able to

CO-1	Describe the basics of CFD and parallel computing and explain the various flow models, its governing equations of fluid motion in differential and integral form. (L3)
CO-2	Compare the physics of the flow with the mathematical behavior of partial differential equations. (L3)
CO-3	Identify and compute the suitable grid generation and transformation techniques for a given problem. (L4)
CO-4	Illustrate the fundamentals of discretization in FVM, FDM and FEM techniques. (L3)
CO-5	Interpret different schemes and their stability in simple CFD applications. (L4)

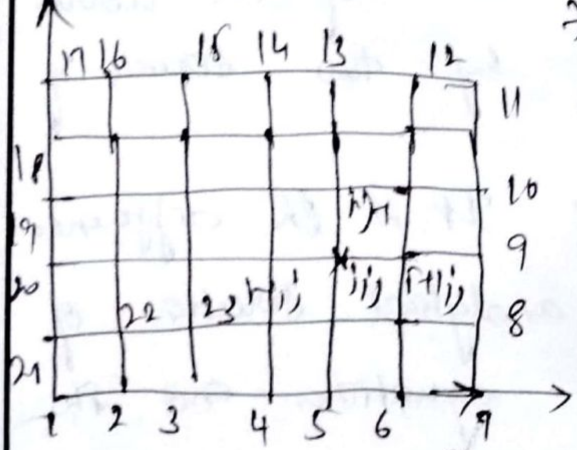
DEPARTMENT: AERONAUTICAL ENGINEERING

Scheme & Solutions- TEST- I/II/III **Date:** 18/1/2022

Semester: VII **Subject Title:** Computational Fluid Dynamics **Subject Code:** 18AE

Question Number	Solution	Marks Allocated
1.	* Explicit method would be numerically unstable if the increment in the marching direction exceeded some prescribed value. The prescription for this maximum allowable value comes in principle from a formal stability analysis of the governing equations in finite-difference form.	-2-
	* Considering one dimensional heat conduction equation as model equation $$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$ $$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \alpha \frac{\left(T_{i+1}^n - 2T_i^n + T_{i-1}^n\right)}{(\Delta x)^2}$$	-2-
	* The numerical solution of the above equation is influenced by two source of error. → Discretisation error: It is the difference between the exact analytical solution of the partial differential equations and the exact solution of the corresponding difference equation.	-2-

Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE72

Question Number	Solution	Marks Allocated
	→ Round off errors: The numerical error introduced after a repetitive number of calculations in which the computer is constantly rounding the numbers to some significant figure	-2-
	If, A - analytical solution of partial differential equation D - Exact solution of difference equation N - numerical solution from a real computer with finite accuracy.	-2-
	Discretization error = A - D Round off error = ϵ = N - D	10
2.	Relaxation Technique:	
	 * It is a finite difference method particularly suited for the solution of elliptic partial differential equations	-2-
	Low speed, subsonic inviscid flow is governed by elliptic pde. and these	

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Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE7

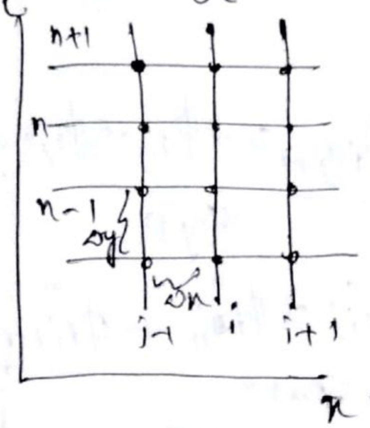
Question Number	Solution	Marks Allocated
	* Relaxation technique can be either explicit or implicit. Consider an inviscid, incompressible, two dimensional irrotational flow.	-2-
	$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$	
	$$\frac{\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}}{(\Delta x)^2} + \frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{(\Delta y)^2} = 0.$$	
	$$\phi_{i,j}^{n+1} = \frac{(\Delta x)^2 (\Delta y)^2}{2\Delta y^2 + 2\Delta x^2}\left[\frac{\phi_{i+1,j}^n + \phi_{i-1,j}^n}{(\Delta x)^2} + \frac{\phi_{i,j+1}^n + \phi_{i,j-1}^n}{\Delta y^2}\right]$$	
	$$\phi_{21}^{n+1} = \frac{\Delta x^2 \Delta y^2}{2\Delta y^2 + 2\Delta x^2}\left[\frac{\phi_{22}^n + \phi_{20}}{(\Delta x)^2} + \frac{\phi_{24}^n + \phi_2}{(\Delta y)^2}\right]$$	-2-
	$$\phi_{22}^{n+1} = \frac{\Delta x^2 \Delta y^2}{2\Delta y^2 + 2\Delta x^2}\left[\frac{\phi_{23}^n + \phi_{21}^{n+1}}{\Delta x^2} + \frac{\phi_{25}^n + \phi_3}{\Delta y^2}\right]$$	
	The unknown ϕ's at iteration n+1 are progressively calculated along a given horizontal line sweeping from left to right. This approach is called Gauss-Seidel method	-2-
	$$\phi_{i,j}^{n+1} = \phi_{i,j}^n + \omega\left(\overline{\phi_{i,j}^{n+1}} - \phi_{i,j}^n\right)$$	-2-
	$\omega > 1$ – successive overrelaxation	
	$\omega < 1$ – under relaxation	10.

Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE72

Question Number	Solution	Marks Allocated

3\. * Considering an unsteady, one dimensional heat conduction equation with constant thermal diffusivity — 2 —

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$



An unsteady thermal conduction is a parabolic partial differential equation which lends they to a marching solution with respect to time t. — 2 —

$$* \left(\frac{\partial T}{\partial t}\right)_i^n = \frac{T_i^{n+1} - T_i^n}{\Delta t} - \left(\frac{\partial^2 T}{\partial t^2}\right)\frac{\Delta t}{2} + \ldots$$

— 2 —

$$\left(\frac{\partial^2 T}{\partial x^2}\right)_i^n = \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2} - \left(\frac{\partial^4 T}{\partial x^4}\right)\frac{\Delta x^2}{12} + \ldots$$

$$\frac{\partial T}{\partial t} - \alpha \frac{\partial^2 T}{\partial t^2} = 0.$$

$$\underbrace{\frac{\partial T}{\partial t} - \alpha \frac{\partial^2 T}{\partial x^2}}_{PDE} = 0 = \underbrace{\frac{T_i^{n+1} - T_i^n}{\Delta t} - \alpha \frac{(T_{i+1}^n - 2T_i^n + T_{i-1}^n)}{\Delta x^2}}_{\text{Difference equation}} +$$

— 2 —

$$\underbrace{\left[- \left(\frac{\partial^2 T}{\partial t^2}\right)^n \frac{\Delta t}{2} + \alpha \left(\frac{\partial^4 T}{\partial x^4}\right)_i^n \frac{\Delta x^2}{12} + \ldots \right]}_{\text{Truncation error}}$$

— 2 —

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \alpha \frac{(T_{i+1}^n - 2T_i^n + T_{i-1}^n)}{\Delta x^2}$$

10.

Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE72

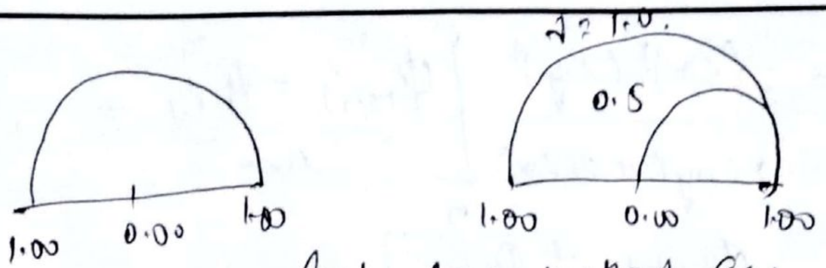
Question Number	Solution	Marks Allocated
A.	Lax Wendroff Technique: It is an explicit finite-difference method particularly suited to marching solution.	-2-
	Consider an unsteady, two-dimensional inviscid flow. The governing Euler equations are re-arranged in nonconservation form and are as obtained below.	
	Continuity, $\frac{\partial \rho}{\partial t} = -\left(\rho \frac{\partial u}{\partial x} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial v}{\partial y} + v\frac{\partial \rho}{\partial y}\right)$	-2-
	x-momentum $\frac{\partial u}{\partial t} = -\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{1}{\rho}\frac{\partial p}{\partial x}\right)$	
	y-momentum, $\frac{\partial v}{\partial t} = -\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{1}{\rho}\frac{\partial p}{\partial y}\right)$	
	Energy, $\frac{\partial e}{\partial t} = -\left(u\frac{\partial e}{\partial x} + v\frac{\partial e}{\partial y} + \frac{p}{\rho}\frac{\partial u}{\partial x} + \frac{p}{\rho}\frac{\partial v}{\partial y}\right)$	
	The above equations are hyperbolic with respect to time and are used to set up a numerical solution using a time-marching approach.	-2-
	* The Lax-Wendroff method is predicated on a Taylor series expansion in time, by choosing any dependent flow variable.	-2-
	$\rho_{i,j}^{t+\Delta t} = \rho_{i,j}^{t} + \left(\frac{\partial \rho}{\partial t}\right)_{i,j}^{t} \Delta t + \left(\frac{\partial^2 \rho}{\partial t^2}\right)_{i,j}^{t} \frac{\Delta t^2}{2} + \cdots$	

Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE72

Question Number	Solution	Marks Allocated
	$\left(\frac{\partial \rho}{\partial t}\right)^t_{i,j} = -\left(\rho^t_{i,j} \frac{u^t_{i+1,j} - u^t_{i-1,j}}{2\Delta x} + u^t_{i,j} \frac{\rho^t_{i+1,j} - \rho^t_{i-1,j}}{2\Delta x} + \rho^t_{i,j} \frac{v_{i,j+1} - v_{i,j-1}}{2\Delta y} + v_{i,j} \frac{\rho_{i,j+1} - \rho_{i,j-1}}{2\Delta y}\right)$	-2-
		10
5.	**Mid point leap frog method:-**	
	* First order schemes are not used to solve PDEs because of their inherent accuracy. The leap frog method is the simplest second order accurate method. When applied to the first order wave equation, this explicit one-step three time level scheme becomes	-2-
	$\frac{u_j^{n+1} - u_j^{n-1}}{2\Delta t} + c\,\frac{u_{j+1}^n - u_{j-1}^n}{2\Delta x} = 0$	-2-
	$\lvert \nu \rvert \le 1,\ u_t + c u_x = \frac{c(\Delta x)^2}{6}(\nu^2 - 1) u_{xxx} - \frac{c(\Delta x)^4}{120}(9\nu^4 - 10\nu^2 + 1) u_{xxxxx} + \ldots$	
	* Amplification factor,	-2-
	$G = \pm 1 (1 - \nu^2 \sin^2\beta)^{1/2} - i\nu \sin\beta$	
	$\frac{\phi}{\phi_e} = \frac{\tan^{-1}\left[-\nu \sin\beta / \pm (1 - \nu^2 \sin^2\beta)^{1/2}\right]}{-\beta\nu}$	-2-

Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE72

Question Number	Solution	Marks Allocated



— 2 —

Advantages of leap frog method are:
– Initial conditions must be specified at two time levels, which is overcome by using a two time level scheme for the first time step.

10

6. Alternating Direct Implicit method:

Consider a model equation based on the unsteady, two-dimensional heat conduction in two spatial dimensions equation

— 2 —

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

$$\frac{T_{i,j}^{n+1} - T_{i,j}^{n}}{\Delta t} = \alpha \frac{\frac{1}{2}\left(T_{i+1,j}^{n+1} + T_{i+1,j}^{n}\right) + \frac{1}{2}\left(-2T_{i,j}^{n+1} - 2T_{i,j}^{n}\right) + \frac{1}{2}\left(T_{i-1,j}^{n+1} + T_{i-1,j}^{n}\right)}{\Delta x^2}$$

$$+ \alpha \frac{\frac{1}{2}\left(T_{i,j+1}^{n+1} + T_{i,j+1}^{n}\right) + \frac{1}{2}\left(-2T_{i,j}^{n+1} - 2T_{i,j}^{n}\right) + \frac{1}{2}\left(T_{i,j-1}^{n+1} + T_{i,j-1}^{n}\right)}{\Delta y^2}$$

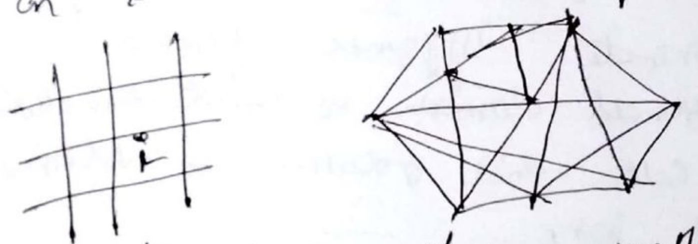
— 2 —

* The relaxation technique is an iterative method, $\phi_{i,j}$ is unknown

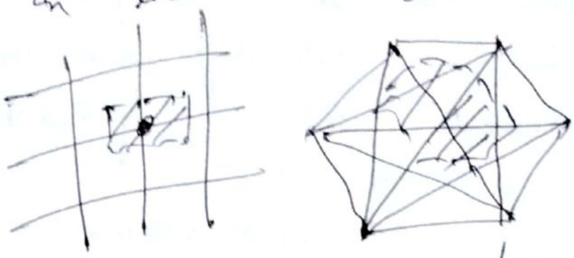
Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE72

Question Number	Solution	Marks Allocated
	$\phi_{i,j}^{n+1} = \frac{(\Delta x)^2(\Delta y)^2}{2(\Delta y)^2+2(\Delta x)^2}\left[\frac{\phi_{i+1,j}^{n} - \phi_{i-1,j}^{n}}{\Delta x^2} + \frac{\phi_{i,j+1}^{n} + \phi_{i,j-1}^{n}}{\Delta y^2}\right]$	
	$\phi_{21}^{n+1} = \frac{(\Delta x)^2 \Delta y^2}{2\Delta y^2 + 2\Delta x^2}\left[\frac{\phi_{22}^{n} + \phi_{20}^{n}}{\Delta x^2} + \frac{\phi_{24}^{n} + \phi_{2}}{\Delta y^2}\right]$	2
	$\frac{T_{i,j}^{n+1/2} - T_{i,j}^{n}}{2} = \alpha\frac{T_{i+1,j}^{n+1/2} - 2T_{i,j}^{n+1/2} + T_{i-1,j}^{n+1/2}}{(\Delta x)^2} + \alpha\frac{T_{i,j+1}^{n} - 2T_{i,j}^{n} + T_{i,j-1}^{n}}{(\Delta y)^2}$	2
	$A T_{i-1,j}^{n+1/2} - B T_{i,j}^{n+1/2} + A T_{i+1,j}^{n+1/2} = K_i$	
	$A = \frac{\alpha\Delta t}{2\Delta x^2}$ $B = 1 + \frac{\alpha\Delta t}{(\Delta x)^2}$	
	$K_i = -T_{i,j}^{n} - \alpha\frac{\Delta t}{2\Delta y^2}(T_{i,j+1} - 2T_{i,j} + T_{i,j-1})^{n}$	2
	$C = \frac{\alpha\Delta t}{2\Delta y^2}$ $D = 1 + \frac{\alpha\Delta t}{\Delta y^2}$	
		10.

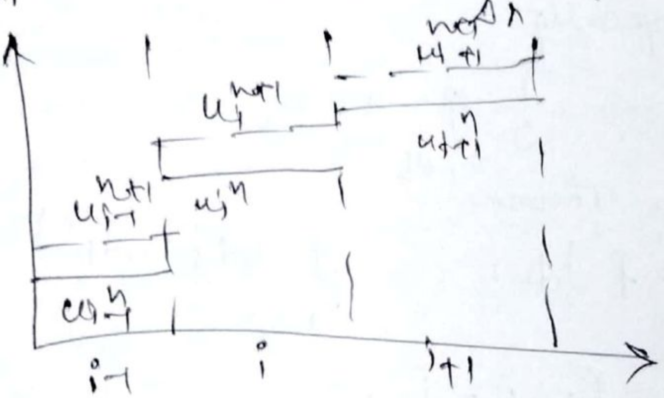
Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE72

Question Number	Solution	Marks Allocated
7.	Cell centred finite volume method:	
	- Straight forward choice - grid cell itself is considered as a finite volume	- 1 -
	- Cell centroids are the control points	
	- Ideally suited for handling grids with	
	- mixed elements considering of hexahedra, prism, pyramid and tet cells.	- 1 -
	- arbitrary polyhedral / polygonal volumes	- 1 -
	- adaptive meshes with hanging nodes	
	- Boundary condition is satisfied through interfacial flux calculation at the faces falling on the domain boundary	- 1 -
		- 1 -
	Cell vertex finite volume method:	
	- Mesh / grid nodes / vertices are the control points	- 1 -
	- Dual finite volume is constructed around the mesh nodes	
	- popularly used around unstructured grid	
	- Known to be advantageous for unstructured triangular / tetrahedral grids for efficient and simple discretization for the viscous fluxes	- 1 -
	- Not suitable for grids with	
	- mixed elements considering of hexahedra, prism, pyramid and tet cells	- 1 -
	- Arbitrary polyhedral volumes	

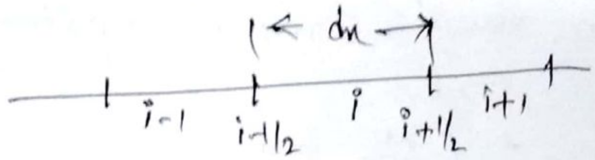
Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE72

Question Number	Solution	Marks Allocated
	- Can lead to unacceptable skewed local volumes	
	- Boundary treatment can be enforced using Dirichlet condition since the mesh vertices fall on the domain boundaries	-1-
		-1-
8.	Finite difference and finite volume method	
	- Finite difference method discretization is based upon the differential form of the PDE to be solved. Each derivative is replaced with an approximate difference formula.	-2-
	- The computational domain is usual discretized into hexahedral cells and solution is obtained at each nodal point.	-1-
	- The discretization results in a system of equations of the variable at nodal points.	-1-
	- They have a discrete representation of the solution	-1-
	FVM: - is a method of representing and evaluating partial differential equation in the form of algebraic equations	-2-
	- In FVM, volume integrals, volume integral in a partial differential equation that contain a divergence term converted to surface integrals.	-1-

Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE72

Question Number	Solution	Marks Allocated
	- Uses divergence theorem for conservation	-1-
	- useful for problems with body fitted co-ordinate system	-1-
		10
9.	Numerical viscosity & Artificial Dissipation	
	- Consider 1D linear convective equation with wave speed $c > 0$.	
	$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0.$	-2-
	$u_i^{n+1} = u_i^n - \frac{c\Delta t}{\Delta x}\left(u_i^n - u_{i-1}^n\right)$	
		-2-
	- Introduces artificial diffusion due to numerical viscosity.	-2-
	- First order methods are diffusive in nature & smear the flow features extensively.	
	- practically, a second order method is used for industrial application because of stability consideration	-2-
	- use of flux limiters	-2-
	- called as high resolution scheme	10.

Subject Title: Computational Fluid Dynamics **Subject Code:** 18AE72

Question Number	Solution	Marks Allocated
10.	Flux vector Splitting method:	
	Considering 1D Conservation law: $\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = 0$	-2-
	Consider a 1D grid system 	
	$\int\left(\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x}\right)dx = 0 \quad \int \frac{\partial U}{\partial t}dx + \int \frac{\partial F}{\partial x}dx = 0$	-2-
	Cell average quantity $\bar{U} = \frac{1}{\Delta x}\oint_{i+1/2}^{i+1/2} U dx$	
	Gauss Divergence Theorem $\int_{\Omega}(\nabla \cdot \vec{F})d\Omega = \oint_{\Gamma}(d\vec{F}\cdot d\vec{\Gamma})$	-2-
	$\int \frac{\partial F}{\partial x}dx = F_{i+1/2} - F_{i-1/2}$	
	$\frac{d(\bar{U}\Delta x)}{dt} + F_{i+1/2} - F_{i-1/2} = 0.$	
	$\frac{d\bar{U}}{dt} + \frac{F_{i+1/2} - F_{i-1/2}}{\Delta x} = 0$	-2-
	Explicit method: $f = f(U^n)$	
	Implicit method - $f = f(U^{n+1})$	-2-
	$\frac{U^{n+1} - U^n}{\Delta t} + \frac{F_{i+1/2} - F_{i-1/2}}{\Delta x} = 0$	
		10

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