Module-2 [Question Bank]

① Find an approximate real root of the equation $x^3-3x+4=0$ using the method of false position, correct to three decimal places which lie b/w -3 and -2. Carry out three iterations.

$f(x) = x^3-3x+4$

The root lies in the interval -3 & -2

$a=-3$, $b=-2$

$f(-3) = -27+9+4 = -14$

$f(-2) = -8+6+4 = 2$

Ist approximation

$$x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)} \Rightarrow \frac{-3(2)-(-2)(-14)}{2+14} = \frac{-6-28}{16}$$

$$x_1 = \frac{-34}{16} = -2.125$$

$$f(x_1) = (-2.125)^3 - 3(-2.125)+4$$
$$= -9.595 + 6.375 + 4$$
$$= 0.78$$

IInd approximation.

$$x_2 = \frac{x_1 f(b) - bf(x_1)}{f(b)-f(x_1)} \Rightarrow \frac{-2.125(2)-(-2)(0.78)}{2-0.78}$$

$$x_2 = \frac{-4.25+1.56}{1.22} = -2.204$$

$$f(x_2) = (-2.204)^3 - 3(-2.204)+4$$
$$= -0.094$$

IIIrd approximation

$$x_3 = \frac{x_2 f(b) - bf(x_2)}{f(b)-f(x_2)} = \frac{-2.204(2)-(-2)(-0.094)}{2+0.094}$$

$\boxed{x_3 = -2.194}$ ∴ Therefore the required root is $\boxed{-2.194}$

2. Compute real root of $x\log_{10}x = 1.2$ using Regula falsi method lies between (2.7, 2.8) Correct to three decimal places of accuracy.

Let $f(x) = x\log_{10}x - 1.2$

The root lies between 2.7 and 2.8

$a = 2.7 \qquad b = 2.8$

$f(2.7) = 2.7\log 2.7 - 1.2 = -0.035$

$f(2.8) = 2.8\log 2.8 - 1.2 = 0.052$

I^{st} approximation.

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} \Rightarrow \frac{2.7(0.052) - 2.8(-0.035)}{0.052 + 0.035}$$

$x_1 = 2.7403$

$f(x_1) = 2.740\log 2.740 - 1.2$

$= -0.0005$

II^{nd}

$$x_2 = \frac{x_1 f(b) - bf(x_1)}{f(b) - f(x_1)} = \frac{2.740(0.052) - 2.8(0.0005)}{0.052 + 0.0005}$$

$x_2 = 2.7405$

III^{rd}

$\boxed{f(x_2) = -0.0005}$

$$x_3 = \frac{x_2 f(b) - bf(x_2)}{f(b) - f(x_2)} = \frac{2.7405(0.052) - 2.8(-0.0005)}{0.052 + 0.0005}$$

$\boxed{x_3 = 2.740}$

$\therefore$ The required root is 2.740

3. find an approximate real root of the equation $\cos x = 3x - 1$ using false position method and carry out 4 iterations.

Let $f(x) = \cos x - 3x + 1$

$f(0) = 1 + 1 = 2 > 0$

$f(1) = 0.9999 - 3 + 1 = -1.0001 < 0$

$f(2) = 0.9993 - 5 = -4.0006 < 0$

$f(-1) = 0.9998 + 4 = 4.9998 > 0$

The required root lies between 0 & 1 (0, 1)

Let $a = 0 \quad b = 1$

I^{st} approximation

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{0(-1.0001) - 1(2)}{-1.0001 - 2} = 0.6666$$

$f(x_1) = \cancel{0.6666(-1.0001) - 1(}$

$= \cos(0.6666) - 3(0.6666) + 1$

$= 0.0001$

II approximation

$$x_2 = \frac{0.6666(-1.0001) - (0.0001)}{-1.0001 - 0.0001} = 0.6665$$

$f(x_2) = \cos(0.6665) - 3(0.6665) + 1$

$= 0.0004$

$$x_3 = \frac{x_2 f(b) - b f(x_2)}{f(b) - f(a)} = \frac{0.6665(-1.0001) - (0.0004)}{-1.0001 - 0.0004}$$

$x_3 = 0.6666$

$f(x_3) = 0.0001$

$\boxed{x_4 = 0.666}$

$\therefore$ The required root is 0.666

find an approximate real root of the equation $x\sin x + \cos x = 0$ using Newton-Raphson method near $x = \pi$. Carry out iterations up to four decimal places of accuracy.

$\pi = 3.1415$

Let $f(x) = x\sin x + \cos x$

$x_0 = \pi$

$f'(x) = x\cos x + \sin x - \sin x$

$x\cos x$.

I[st] approximation

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$f(\pi) = \pi.\sin\pi + \cos\pi$
$= -1$

$$x_1 = \pi - \frac{f(\pi)}{f'(\pi)}$$

$f'(\pi) = \pi\cos\pi$
$= -3.1415$

$$x_1 = 3.14 - \frac{(-1)}{-3.1415}$$

$$\boxed{x_1 = 2.8231}$$

II[nd] approximation

$= f(2.8231)$

$f() = -0.0656$

$f'(2.8231) = -2.6811$

$$x_2 = 2.8231 - \frac{(-0.0656)}{-2.6811}$$

$$x_2 = 2.7986$$

III[rd] approximation

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$f(x_2) = 2.7986\sin 2.7986 + \cos$

$f'(x_2)\ 2.7986\cos(2.7986)$

$$2.7986 - \frac{(-0.0005)}{(-2.6355)}$$

$$\boxed{x_3 = 2.7984}$$

$\therefore$ the required root is 2.7984

5. Use Newton Rapson method to find real root of eqn $x\log_{10} x = 1.2$ near (2,3) Carry out three iterations.

$f(x) = x\log_{10} x = 1.2 \quad f'(x) = 1 + \log x$

$f(2) = 2\log_{10} 2 - 1.2 = -0.5979$

$f(3) = 3\log_{10} 3 - 1.2 = 0.2313$

The required root lies between (2,3)

I^{st} approximation

let $x_0 = 2$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ where $f(x_0) = -0.5979$, $f'(x_0) = 0.7350$

$2 - \frac{(-0.5979)}{\cancel{0.7350}\ 0.7353}$

$x_1 = x_0 - \frac{x_0\log_{10} x_0 - 1.2}{\log_{10} e + \log_{10} x_0}$ $\quad f'(x) = \log_{10} e + \log_{10} x_0$

$\boxed{x_1 = 2.8131}$

2nd approximation

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_2 = 2.8131 - \frac{0.0636}{0.8834}$

$\boxed{x_2 = 2.7411}$

III rd approximation

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$x_3 = 2.7411 - \frac{0.0003}{0.8722}$

$\boxed{x_3 = 2.7407}$

$\therefore$ The required root is $\boxed{2.740}$

7. From the following table, estimate the number of students who got marks between 40 to 45

Marks:	30-40	40-50	50-60	60-70	70-80
No of Students:	31	42	51	35	31

Soln: $f(x)$ be the number of students less than x marks

x	40	50	60	70	80
No of students less than x	31	73	124	159	190

x	y_0	Δy_0	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
40	$31 = y_0$				
		42			
50	73		9		
		51		-25	
60	124		-16		37
		35		12	
70	159		-4		
		31			
80	190				

$$r = \frac{x - x_0}{h} = \frac{45-40}{10} = \frac{5}{10} = 0.5$$

$$y(45) = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 \ldots$$

$$y(45) = 31 + 0.5(42) + \frac{0.5(-0.5)(9)}{2} + \frac{0.5(-0.5)(-1.5)(-25)}{3!} + \frac{0.5(0.5)(-1.5)(-2.5)(37)}{4!}$$

$$y(45) = 48.$$

No of students who scored between 40 to 45

$= y(45) - y(40)$

$= 48 - 31$

$\boxed{= 17}$

8. Given $\sin 45^\circ = 0.7071$, $\sin 50^\circ = 0.7660$, $\sin 55^\circ = 0.8192$, $\sin 60^\circ = 0.8660$. find $\sin 48^\circ$ using Newton's forward interpolation formula.

Given:

	x	y	Δy_0	$\Delta^2 y$	$\Delta^3 y$
x_0	$\sin 45^\circ$	0.7071			
			0.0589		
x_1	50°	0.7660		-0.0057	
			0.0532		-0.0007
x_2	55°	0.8192		-0.0064	
			0.0468		
x_3	60°	0.8660			

$$r = \frac{x - x_0}{h} = \frac{48 - \cancel{40}\,45}{5} = \frac{\cancel{+12}\,3}{5} = \cancel{-2.4}\ 0.6$$

$\boxed{r = 0.6}$

$$Y(48^\circ) = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3}\Delta^3 y_0$$

$$Y(48)' = 0.7071 + 0.6(0.0589) + \frac{0.6(-0.4)\,-0.0057}{2} + 0.6(-0.4)(-1.4)(-0.0007)/3\times 2.$$

$$\boxed{Y(48) = 0.7435}$$

9. The area of a circle of diameter D is given for the following values.

D :	80	85	90	95	100
A :	5026	5674	6362	7088	7854

use appropriate interpolation formula to find area of circle at D = 105

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
80	5026				
		648			
85	5674		40		
		688		-2	
90	6362		38		4
		726		2	
95	7088		40		
		766			
100	7854				

$y(105) =$ $\quad r = \frac{x - x_n}{h} = \frac{105 - 100}{5} = \frac{5}{5} = 1$

Newton's Backward Interpolation formula.

$$y = y_n + r\nabla y_n + \frac{r(r+1)}{2}\nabla^2 y_n + \frac{(r(r+1)(r+2))}{3!}\nabla^3 y_n + \cdots$$

$$y(105) = 7854 + 766 + \frac{2(40)}{2} + \frac{2.3.2}{6} + \frac{2.3.4(4)}{24}$$

$$\boxed{y(105) = 8666}$$

10. Given $f(40) = 184$, $f(50) = 204$, $f(60) = 226$; $f(70) = 250$ $f(80) = 276$, $f(90) = 304$, find $f(38)$ and $f(85)$ using suitable interpolation. formule.

x	y	Δy_0	$\Delta^2 y$
40	184		
		20	
50	204		2
		22	
60	226		2
		24	
70	250		2
		26	
80	276		2
		28	
90	304		

$r = \frac{x - x_0}{h} = \frac{38 - 40}{10}$

$r = \frac{-2}{10} = -0.2$

To find $f(38)$

Newton's forward Interpolation formula.

$$y = y_0 + r\Delta y_0 + \frac{r(r-1)}{2}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0.$$

$$y(38) = 184 + (-0.2)20 + \frac{(-0.2)(-1.2)(\cancel{2})}{\cancel{2}}$$

$$\boxed{y(38) = 180.24}$$

$f(85) \Rightarrow \quad r = \frac{x - x_n}{h} = \frac{85 - 90}{10} = \frac{-5}{10} = -0.5$

$$y = 304 + (-0.5)(28) + \frac{(-0.5)(-1.5)\cancel{2}}{\cancel{2}}$$

$$\boxed{y(85) = 290.75}$$

11. For the following data Calculate the difference & obtain backward difference interpolation polynomial. Hence find $f(0.35)$

x	y	Δy	$\Delta^2 y$
0.1	1.40		
		0.16	
0.2	1.56		0.04
		0.2	
0.3	1.76		0.04
		0.24	
0.4	2.0		0.04
		0.28	
0.5	2.28		

$$r = \frac{x - x_n}{h} = \frac{0.35 - 0.5}{0.1} = -1.5$$

$$y(0.35) = 2.28 + (-1.5)(0.28) + \frac{(-1.5)(-0.5)(0.04)}{2}$$

$$\boxed{y(0.35) = 1.875}$$

12. For the following data find $f(9)$ using Newton's divided difference formula.

x:	5	7	11	13	17
$f(x)$:	150	392	1452	2366	5202

Newton divided difference Interpolation

	x	$f(x)$	Ist D.D	IInd D.D	IIIrd D.D
x_0	5	150			
			$f(x_0,x_1)=121$		
x_1	7	392		$f(x_0,x_1,x_2)=24$	
			$f(x_1,x_2)=265$		$f(x_0,x_1,x_2,x_3)=1$
x_2	11	1452		$f(x_1,x_2,x_3)=32$	
			$f(x_2,x_3)=457$		$f(x_1,x_2,x_3,x_4)=1$
x_3	13	2366		$f(x_2,x_3,x_4)=42$	
			$f(x_3,x_4)=709$		
x_4	17	5202			

$$f(x) = 5 + 4(121) + 4.2.24 + 4.2(-2).(1)$$

$$\boxed{f(x) = 810}$$

13. Construct an interpolating polynomial for the data given below using Newton's generalized interpolation formula

x:	2	4	5	6	8	10
y:	10	96	196	350	868	1746

and hence find y at $x=7$

	x	y	I st D.D	II nd D.D	III rd D.D
x_0	2	10	$f(x_0,x_1)=43$	$f(x_0,x_1,x_2)=19$	$f(x_0,x_1,x_2,x_3)=2$
x_1	4	96	$f(x_1,x_2)=100$	$f(x_1,x_2,x_3)=27$	$f(x_1,x_2,x_3)=2$
x_2	5	196	$f(x_2,x_3)=154$	$f(x_2,x_3,x_4)=35$	$f(x_2,x_3,x_4)=2$
x_3	6	350	$f(x_3,x_4)=259$	$f(x_3,x_4,x_5)=45$	
x_4	8	868	$f(x_4,x_5)=439$		
x_5	10	1746			

$$f(x)=f(x_0)+(x-x_0)f(x_0,x_1)+(x-x_0)(x-x_1)f(x_0,x_1,x_2)+(x-x_0)(x-x_1)(x-x_2)f(x_0,x_1,x_2,x_3)$$

$$y=10+(x-2).43+(x-2)(x-4)19+(x-2)(x-4)(x-5).2$$

$$y=10+43x+86+19x^2-114x+152+2x^3-22x^2+76x-80$$

$$y=2x^3-3x^2+5x-4$$

This is the required interpolating polynomial

finding at $x=7$

$$y=2.343-3.49+5.7-4$$

$$\boxed{y=570}$$

14. For the following data find $f(9)$ using Lagrange's interpolation formula

x:	5	7	11	13	17
$f(x)$:	150	392	1452	2366	5202

$$y = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_3$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} y_4$$

$$y = \frac{2.-2.-4.-8}{-2.-6.-8.-12}(150) + \frac{4.-2.-4.-8}{2.-4.-6.-10}(392) + \frac{4.2.-4.-8}{6.4.-2.-6}(1452) + \frac{4.2.-2.-8}{8.6.2.-4}(2366) + \frac{4.2.-2.-4}{12.10.6.4}(5202)$$

$$y = -\frac{150}{9} + \frac{8.392}{15} + \frac{8}{9}(1452) + \left(-\frac{2366}{3}\right) + \frac{5202}{45}$$

$$\boxed{y_{(9)} = 810}$$

15.

(15) Use Lagrange's interpolation formula to find the polynomial for the data

x :	0	1	3	4
y :	-12	0	6	12

hence find y at $x = 2$

$$y(2) = \frac{(x-1)(x-3)(x-4)}{(0-1)(0-3)(0-4)}(-12) + \frac{(x-0)(x-3)(x-4)}{(1-0)(1-3)(1-4)}(0)$$

$$\frac{(x-0)(x-1)(x-4)}{(3-0)(3-1)(3-4)}(6) + \frac{(x-0)(x-1)(x-3)}{(4-0)(4-1)(4-3)}(12)$$

$$y = \frac{1.-1.-2.-12}{-1.-3.-4} + \frac{2.1.-2}{3.2.-1}(6) + \frac{2.1.-1}{4.3.1}(12)$$

$$y = \cancel{2} + 4 \cancel{-2}$$

$$\boxed{y = 4}$$

16. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using Trapezoidal, simpsons $\frac{1}{3}$rd and Simpsons $3/8$th rules by taking 6 equal parts.

Comparing with given integral

$$\int_0^6 \frac{dx}{1+x^2} = \int_a^b y \cdot dx.$$

$y = \frac{1}{1+x^2}$ $\quad a = 0 \quad b = 6 \quad n = 6$

$$h = \frac{b-a}{n} = \frac{6-0}{6} = 1$$

The values of x are:

x_0	x_1	x_2	x_3	x_4	x_5	x_6
0	1	2	3	4	5	6

$y_0 = \frac{1}{1+x^2} = 1$

$y_1 = \frac{1}{2} = 0.5$

$y_2 = 0.2$

$y_3 = 0.1$

$y_4 = 0.0588$

$y_5 = 0.0384$

$y_6 = 0.0270$

Trapezoidal method

$$I = \frac{h}{2}[y_0 + y_n] + 2(y_1 + y_2 + y_3 + y_4 + y_5]$$

$$= \frac{1}{2}[[1 + 0.0270] + 2(0.5 + 0.2 + 0.1 + 0.0588 + 0.0384)].$$

$$\boxed{I = 1.4107}$$

Simpson's $\frac{1}{3}$rd rule:

formula is $I = \frac{h}{3}[(y_0+y_n)+4(y_1+y_3+y_5)+2(y_2+y_4)$

$I = \frac{1}{3}[1+0.0270]+4(0.5+0.1+0.0384)+2(0.2+0.0588)$

$\boxed{I = 1.3660}$

Simpsons 3/8th rule:

formula is $I = \frac{3}{8}h(y_0+y_n)+3(y_1+y_2+y_4+y_5)+2(y_3)$

$I = \frac{3}{8}(1+0.0270)+3(0.5+0.2+0.0588+0.0384)+2(0.1)$

$\boxed{I = 1.3569}$

17. Evaluate $\int_0^{\pi/2}\sqrt{\cos\theta}.d\theta$ by using simpsons 3/8th rule by taking 10 ordinates.

Comparing given Integral.

$\int_a^{\pi/2}\sqrt{\cos\theta}.d\theta = \int_a^b y.d\theta.$

$a=0 \quad b=\pi/2 \quad y=\sqrt{\cos\theta} \quad n=9 \quad h=\frac{\pi}{18} \quad 10^\circ$

$x_0 = (0^\circ) \quad y_0 = \sqrt{\cos 0} = 1$

$x_1 = \frac{\pi}{18}(10^\circ) \quad y_1 = \sqrt{\cos(\pi/18)} = 0.9923$

$x_2 = \frac{2\pi}{18}(20^\circ) \quad y_2 = \sqrt{\cos(2\pi/18)} = 0.9693$

$x_3 = \frac{3\pi}{18}(30^\circ) \quad y_3 = \sqrt{\cos(30^\circ)} = 0.9306$

$x_4 = 4\pi/18\,(40^\circ) \quad y_4 = \sqrt{\cos(40^\circ)} = 0.8752$

$x_5 = 5\pi/18\,(50^\circ) \quad y_5 = \sqrt{\cos(50^\circ)} = 0.8017$

$x_6 = 6\pi/18\,(60^\circ) \quad y_6 = \sqrt{\cos(60^\circ)} = 0.7071$

$x_7 = 7\pi/18\,(70^\circ) \quad y_7 = \sqrt{\cos(70^\circ)} = 0.5848$

$x_8 = 8\pi/18\,(80^\circ) \quad y_8 = \sqrt{\cos(80)} = 0.4167$

$x_9 = 9\pi/18\,(90^\circ) \quad y_9 = \sqrt{\cos(90^\circ)} = 0$

formula

$I = \frac{3h}{8}[y_0+y_9] + 3(y_1+y_2+y_4+y_5+y_7+y_8] + 2[y_3+y_6]$

$y_0 = 1$

$\cancel{y_1 = 0.9999}$

$\cancel{y_2} = 0.9999$

$y_3 = \cancel{0.9999}$

$y_4 = \cancel{0.9999}$

$y_5 = \cancel{0.9999}$

$y_6 = \cancel{0.9999}$

$y_7 = \cancel{0.9998}$

$y_8 = \cancel{0.9998}$

$y_9 = \cancel{0}$

$= \frac{30}{8}[1+0] + 3[0.9923+0.9693+0.8752+0.8017+0.5848+0.4167] + 2[0.9306+0.7071]$

$= \frac{3\pi}{18.8}[18.1954]$

$\boxed{I = 1.190}$

18. Estimate the value of $\int_{-1}^{1} 2e^{-3x}$ using trapezoidal, Simpsons 1/3rd rules by taking 9 values.

Comparing given integral

$$\int_{-1}^{1} 2e^{-3x}\,dx = \int_a^b y \cdot dx$$

$a = -1 \quad b = 1 \quad n = 9 \quad y = 2e^{-3x} \quad h = \frac{1+1}{9} = \frac{2}{9} = 0.22$

$x_0 = -1$ $\quad y_0 = 2e^{-3(-1)} = 40.171$

$x_1 = -0.7778$ $\quad y_1 = 20.625$

$x_2 = -0.5556$ $\quad y_2 = 10.5903$

$x_3 = -0.3334$ $\quad y_3 = 5.4376$

$x_4 = -0.1112$ $\quad y_4 = 2.7919$

$x_5 = 0.111$ $\quad y_5 = 1.433$

$x_6 = 0.3332$ $\quad y_6 = 0.7360$

$x_7 = 0.5554$ $\quad y_7 = 0.3779$

$x_8 = 0.7778$ $\quad y_8 = 0.1939$

$x_9 = 1$ $\quad y_9 = 0.0995$

Trapezoidal method

$I = \frac{h}{2}[y_0+y_9] + 2[y_1+y_2+y_3+y_4+y_5+y_6+y_7+y_8]$

$I = \frac{0.22}{2}[40.171+0.0995] + 2[20.625+10.5903+5.4376+2.7919+1.433+0.7360+0.3779+0.1939]$

$\boxed{I = 13.710}$

Simpson 1/3rd rule.

$$I = \frac{h}{3}[y_0 + y_9] + 4[y_1 + y_3 + y_5 + y_7] + 2[y_2 + y_4 + y_6 + y_8]$$

$$= \frac{0.22}{3}[40.171 + 0.0995] + 4[20.625 + 5.4376 + 1.433 + 0.3779]$$
$$2[10.5903 + 2.7919 + 0.7360 + 0.1939]$$

$$\boxed{I = 13.228}$$

19. Use Simpson's 1/3rd rule to find $\int_4^{5.2} \log x . dx$ by taking 7 ordinates.

Comparing with given integral

$$\int_4^{5.2} \log x . dx = \int_a^b y . dx$$

$y = \log x$, $a = 4$ $b = 5.2$ $n = 6$ $h = \frac{b-a}{n} = \frac{1.2}{6} = \underline{\underline{0.2}}$

$x_0 = 4$	$y_0 = \log_e 4 = \cancel{0.602}$ 1.386
$x_1 = 4.2$	$y_1 = \log_e 4.2 = 1.4350$
$x_2 = 4.4$	$y_2 = \log_e 4.4 = 1.4816$
$x_3 = 4.6$	$y_3 = \log_e 4.6 = 1.5260$
$x_4 = 4.8$	$y_4 = \log_e 4.8 = 1.5686$
$x_5 = 5.0$	$y_5 = \log_e 5 = 1.6094$
$x_6 = 5.2$	$y_6 = \log_e 5.2 = 1.6486$

$$I = \cancel{0.22}\frac{h}{3}[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)$$

$$I = \frac{0.2}{3}[1.386 + 1.6486] + 4(1.4350 + 1.5260 + 1.6094)$$
$$+ 2[1.4816 + 1.5686]$$

$$I = \frac{0.2}{3}[27.4166]$$

$$\boxed{I = 1.8277}$$

20. Evaluate $\int_0^6 \frac{x}{1+x^2}\,dx$ by taking 7 ordinates using Trapezoidal rule.

Comparing the Integral.

$$\int_0^6 \frac{x}{1+x^2}\,dx = \int_a^b y\,dx$$

$a=0 \quad b=6 \quad n=6 \quad h=1$

$x_0 = 0$	$y_0 = 0$
$x_1 = 1$	$y_1 = 0.5$
$x_2 = 2$	$y_2 = 0.4$
$x_3 = 3$	$y_3 = 0.3$
$x_4 = 4$	$y_4 = 0.2352$
$x_5 = 5$	$y_5 = 0.1923$
$x_6 = 6$	$y_6 = 0.1621$

Trapezoidal rule.

$$I = \frac{h}{2}[y_0 + y_n] + 2(y_1+y_2+y_3+y_4+y_5)$$

$$I = \frac{1}{2}[0+0.1621] + 2(0.5+0.4+0.3+0.2352+0.1923)$$

$$\boxed{I = 1.7085}$$

21. Evaluate $\int_0^5 \frac{dx}{4x+5}$ by simpson's $(1/3)^{rd}$ rule, dividing the interval into 10 equal parts.

Comparing the Integral

$$\int_0^5 \frac{dx}{4x+5} = \int_a^b y\cdot dx \qquad a=0 \quad b=5 \quad n=10 \quad h=\underline{0.5}555$$

$$y = \frac{1}{4x+5}$$

x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	
0.2	0.1428	0.1111	0.0909	0.0769	0.0666	0.0588	0.0526	0.04761	0.0434	0.04

$$I = \frac{h}{3}[(y_0+y_{10}) + 4(y_1+y_3+y_5+y_7+y_9) + 2(y_2+y_4+y_6+y_8)$$

$$= \frac{0.5}{3}[(0.04) + 4(0.1428+0.0909+0.0666+0.0526+0.0434) + 2(0.1111+0.0769+0.0588+0.0476)]$$

$$\boxed{I = 0.4023}$$