

## Department of Mathematics <u>QUESTION BANK</u>

SUBJECT TITLE	Mathematics-2 for CSE Stream		
SUBJECT TYPE	CORE		
SUBJECT CODE	BMATS201		
ACADEMIC YEAR	2024-25	ВАТСН	2024-2027
SCHEME	CBCS 2024		
SEMESTER	П		

	Module -1			
Q. No.	Questions	Blo om 's LL	COs	
1	Find the angle between normal to the surface $xy = z^2$ at the points (4,1,2) and (3,3,-3).	L2	CO1	
2	Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - 3 = z$ at the point (2, -1,2)	L2	CO1	
3	If $\vec{F} = grad(x^3 + y^3 + z^3 - 3xyz)$ , find $div\vec{F}$ and $Curl\vec{F}$	L2	CO1	
4	If $\vec{F} = \nabla(xy^3z^2)$ , find $div\vec{F}$ and $Curl\vec{F}$ at the point $(1, -1, 1)$ .	L2	CO1	
5	Find the directional derivative of $\varphi = 4xz^3 - 3x^2y^2z$ at $P(2, -1, 2)$ along $2\hat{i} - 3\hat{j} + 6\hat{k}$	L2	CO1	
6	Show that $\vec{F} = \frac{x\hat{\imath} + y\hat{\jmath}}{x^2 + y^2}$ is both Solenoidal and irrotational	L3	CO1	
7	Find the constants $a, b \& c$ , such that $\vec{F} = (x + y + az)\hat{\imath} + (bx + 2y - z)\hat{\jmath} + (x + cy + 2z)\hat{k}$ is an irrotational.	L2	CO1	
8	Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $P(2,-1,2)$ along $2\hat{i} - 3\hat{j} + 6\hat{k}$ .	L3	CO1	

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9	Find the directional derivative of $\phi = xy^2 + yz^3$ at (1,2, -1) in the direction normal to the surface $x \log z - y^2 = -4$ at (-1,2,1).	L2	CO1
10	Find the constant 'a' so that $\vec{F} = y(ax^2 + z)i + x(y^2 - z^2)j + 2xy(z - xy)k$ is Solinoidal	L2	CO1
11	If $\vec{F} = (x + y + 1)i + j - (x + y)k$ , show that $\vec{F} . curl \vec{F} = 0$ .	L3	CO1
12	Find the constants <i>a</i> and <i>b</i> such that $\vec{F} = (axy + z^3)i + (3x^2 - z)j + (bxz^2 - y)k$ is irrotational	L3	CO1
13	Analyse the constant <i>a</i> such that $\vec{F} = (axy - z^3)i + (a - 2)x^2j + (1 - a)xz^2k$ is irrotational	L4	CO1
14	Express vector $\vec{A} = zi - 2xj + yk$ in Cylindrical coordinate system.	L1	CO1
15	Show that spherical coordinate system is orthogonal.	L3	CO1
16	Show that cylindrical coordinate system is orthogonal.	L3	CO1
17	Express vector $\vec{A} = zi - 2xj + yk$ in Spherical coordinate system.	L1	CO1
18	Using Mathematical tools, write the code to find $grad(xy^2 + yz^3)$	L5	CO1

	Module -2		
Q. No.	Questions	Blo om 's LL	COs
1	Prove the subspace $W=\{(x, y, z)   x - 3y + 4z = 0\}$ of vector space $\mathbb{R}^3$ is a subspace of $\mathbb{R}^3$ .	L3	CO 2
2	Let $V = R^3$ be a vector space and consider the subset W of V containing the vectors of the form (a, $a^2$ , b) where the second component is the square of the first. Is W a subspace of V.	L2	CO2

3	Find the Basis and dimension of the subspace spanned by the vectors $\{(2,4,2), (1,-1,0), (1,2,1), (0,3,1)\}$ in V <sub>3</sub> (R).	L2	CO2
4	Let $f(x) = 2x^2-5$ and $g(x) = x+1$ , show that the function h(x) =4x^2+3x-7 lies in the subspace span {f, g} of P <sub>2</sub> .	L3	CO2
5	Determine whether the matrix $\begin{bmatrix} -1 & 7 \\ 8 & -1 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ , $\begin{bmatrix} 2 & -3 \\ 0 & 3 \end{bmatrix}$ & $\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$ in the vector space M <sub>22</sub> of 2x2 matrices.	L1	CO2
6	Prove that the transformation $T; R^2 \rightarrow R^2$ defined by $T(x, y) = (3x, x+y)$ is linear. Find the images of the vectors (1, 3) and (-1, 2) under this transformation.	L3	CO2
7	Show that the functions $f(x) = 3x - 2$ and $g(x) = x$ are orthogonal in $P_n$ with inner product $\langle f, g \rangle = \int_0^1 f(x)g(x)dx$ .	L3	CO2
8	Find the kernel and range of the linear operator T (x, y, z) = (x + y, z) of $\mathbb{R}^3 \rightarrow \mathbb{R}^2$	L2	CO2
9	Show that the set S = $\{(1, 2, 4), (1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ is linearly dependent.	L3	CO2
10	Let $P_n$ be the vector space of real polynomial functions of degree $\Box$ n. Show that the transformation $T: P_2 \rightarrow P_1$ defined by $T(ax^2 + bx + c) = (a + b) x + c$ is linear.	L3	CO2
11	Verify the Rank-nullity theorem for the linear transformation T: $V_3(R) \rightarrow V_2(R)$ defined by $T(x, y, z) = (y - x, y - z)$ .	L2	CO2
12	Analyse the matrix of the linear transformation T: $V_2(R) \rightarrow V_3(R)$ such that T (-1, 1) = (-1, 0, 2) and T (2, 1) = (1, 2, 1).	L4	CO2
13	Verify the Rank-nullity theorem for the linear transformation T: $V_3(R) \rightarrow V_3(R)$ defined by $T(x, y, z) = (x+2y-z, y+z, x+y-2z)$ .	L2	CO2
14	Show that the functions $f(t) = 4t+3$ and $g(t) = t^2$ the inner product $\langle f, g \rangle = \int_0^1 f(t)g(t)dt$ . Find $\langle f, g \rangle$ and norm of g.	L3	CO2
15	If $f(t) = t+2$ and $g(t) = 3t-2$ , $h(t)=t^2-2t-3$ and $\langle f, g \rangle = \int_0^1 f(t)g(t)dt$ . the inner product $\langle f, g \rangle = \int_0^1 f(t)g(t)dt$ . Find $\langle f, g \rangle$ , $\langle g, h \rangle$ and norm of g and f.	L2	CO2

16	Prove that the transformation $T; R^2 \rightarrow R^2$ defined by $T(x, y) = (2x+3y, 4x-5y)$ with respect to the basis, $B_1=\{(1,2), (2,5)\}$ of $R^2$ .	L3	CO2
17	Find the Basis and dimension of the subspace spanned by the vectors $\{(1, -2, 3), (1, -3, 4), (-1, 1, -2)\}$ in V <sub>3</sub> (R).	L1	CO2
18	Using Mathematical tools, write the code to Find the dimension of the subspace spanned by the vectors $\{(1,2,3), (2,3,1), (3,1,2)\}$ in $V_3(R)$ .		CO 2

	Module -3				
Q. No.	Questions	Blo om' s LL	COs		
1	i) Find the remainder when $2^{23}$ is divided by 47. (ii) Find the last digit in $7^{118}$ .	L1	CO3		
2	Find the solutions of the linear congruence $11x \equiv 4 \pmod{25}$ .	L2	CO3		
3	Encrypt the message STOP using RSA with key (2537, 13) using the prime numbers 43 and 59.	L5	CO3		
4	Using Fermat's Little Theorem, show that $8^{30}-1$ is divisible by 31.	L3	CO3		
5	Solve the system of linear congruence $x=3(mod5), y=2(mod6), z=4(mod7)$ using Remainder Theorem.	L3	CO3		
6	<ul> <li>i) Find the remainder when 175 × 113 × 53 is divided by 11.</li> <li>ii) (ii) Solve x3+5x+1=0(mod27).</li> </ul>	L3	CO3		
7	Find the least positive values of x such that i) $71=x(mod8)$ ii) 78+x=3(mod5) iii) $89=(x+3)(mod4)$	L2	CO3		
8	Find the remainder when $(349 \times 74 \times 36)$ is divided by 3	L2	CO3		
9	Solve $2x+6y=1(mod7)$ , $4x+3y=2(mod7)$	L3	CO3		
10	Find the last digit of $7^{2013}$ ii) Find the last digit of $13^{37}$	L1	CO3		
11	Find the remainder when the number $2^{1000}$ is divided by 13	L1	CO3		
12	Find the remainder when 14! is divided by 17	L1	CO3		

13	Find the least positive values of x' such that i) $78 + x=3 \pmod{5}$ ii) $89 = (x+3) \pmod{4}$	L2	CO3
14	Find the solution of the linear congruence $14x = 12 \pmod{18}$	L2	CO3
15	<ul> <li>i)Find the remainder when 2<sup>23</sup> is divided by 47.</li> <li>ii) Find the last digit in 7<sup>118</sup></li> </ul>	L2	CO3
16	<ul> <li>i)Find the remainder when 175x115x53 is divided by 11.</li> <li>ii) Solve x<sup>2</sup>+2x -3 =0 (mod 9)</li> </ul>	L2	CO3
17	Using Mathematical tools, write the code to Find the solutions of the linear congruence $11x = 4 \pmod{25}$ .	L5	CO3

	Module -4				
Q. No.	Questions	Blo om 's LL	COs		
1	Find an approximate real root of the equation $x^3 - 3x + 4 = 0$ using the method of false position, correct to three decimal places which lie between -3 and -2. Carry out three iterations.	L2	CO 4		
2	Compute real root of $x \log_{10} x = 1.2$ using Regular falsi method lies between (2.7,2.8) Correct to three decimal places of accuracy.	L2	CO4		
3	Find an approximate real root of the equation $\cos x = 3x - 1$ using False position method and carry out 4 iterations.	L2	CO4		
4	Find an approximate real root of the equation $x \sin x + \cos x = 0$ using Newton- Raphson method near $x = \pi$ . Carry out iterations up to four decimal places of accuracy.	L2	CO4		
5	Use Newton-Raphson method to find real root of equation $x \log_{10} x = 1.2$ near 2.3. Carry out three iterations.	L3	CO4		
6	Analyse the real root of the equation $\cos x = xe^x$ , by the Newton-Raphson method, correct to four decimal places.	L4	CO4		

	From the following table, estimate the number of students who got marks between 40 to 45		
7	Marks: 30-40 40-50 50-60 60-70 70-80	L3	CO4
	No of Students: 31 42 51 35 31		
8	Given, sin $45^{0}$ = 0.7071, sin $50^{0}$ = 0.7660, sin $55^{0}$ = 0.8192, sin $60^{0}$ = 0.8660, find sin $48^{0}$ using Newton's forward interpolation formula.	L2	CO4
	The area of a circle of diameter D is given for the following values		
9	D: 80 85 90 95 100	L2	CO4
	A: 5026 5674 6362 7088 7854 use appropriate interpolation formula to find area of circle at D=105		
	Given $f(40) = 184$ , $f(50) = 204$ , $f(60) = 226$ , $f(70) = 250$ , $f(80) = 276$ ,		
10	f(90) = 304, find $f(38)$ and $f(85)$ using suitable interpolation formule.	L2	CO4
11	For the following data calculate the difference and obtain backward difference interpolation polynomial. Hence find f(0.35)x:0.10.20.30.40.5f(x):1.401.561.762.02.28	L2	CO4
12	For the following data find f(9) using Newton's divided difference formulax:57111317f(x):150392145223665202	L2	CO4
13	Construct an interpolating polynomial for the data given below using Newton's generalized interpolation formula x: 2 4 5 6 8 10 y: 10 96 196 350 868 1746 and hence find y at x = 7.	L3	CO4
14	For the following data find f(9) using Lagrange's interpolation formulax:57111317f(x):150392145223665202	L2	CO4
15	Use Lagrange's interpolation formula to find the polynomial for the data x: 0 1 3 4 y: -12 0 6 12 hence find y at x = 2. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using Trapezoidal, Simpsons 1/3 <sup>rd</sup> and Simpsons	L3	CO4
16	Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using Trapezoidal, Simpsons 1/3 <sup>rd</sup> and Simpsons 3/8 <sup>th</sup> rules by taking 6 equal parts.	L3	CO4

17	Evaluate $\int_{0}^{\frac{\pi}{2}} \sqrt{\cos \theta} d\theta$ by using Simpsons 3/8 <sup>th</sup> rule by taking 10 ordinates.	L3	CO4
18	Estimate the value of $\int_{-1}^{1} 2e^{-3x} dx$ using Trapezoidal, Simpsons $1/3^{rd}$ rules by taking 9 values.	L3	CO4
19	Use Simpson's $1/3^{rd}$ rule to find $\int_{4}^{5.2} \log x  dx$ by taking 7 ordinates.	L2	CO4
20	Evaluate $\int_0^6 \frac{x}{1+x^2} dx$ by taking 7 ordinates using Trapezoidal rule.	L3	CO4
21	Evaluate $\int_0^5 \frac{dx}{4x+5}$ by Simpson's $(1/3)^{rd}$ rule, dividing the interval into 10 equal parts.	L3	CO4
22	Using Mathematical tools, write the code to Find an approximate real root of the equation $\cos x = 3x - 1$ using False position method and carry out 4 iterations.		

	Module -5		
Q. No.	Questions	Blo om 's LL	COs
1	Employ Taylors series method to obtain approximate value of y at x=0.2 for $\frac{dy}{dx} = 2y + 3e^x$ , y(0) = 0.	L3	CO 4
2	Solve $\frac{dy}{dx} = e^x - y^2$ , $y(0) = 1$ using Taylor's series method to evaluate y (0.2).	L3	CO 4
3	By using modified Euler's method, obtain solution of the equation $\frac{dy}{dx} = x +  \sqrt{y} $ with initial condition y=1 at x=0 for the range $0 \le x \le 0.4$ with step size of 0.2.	L2	CO 4
4	By using modified Euler's method, Examine y(0.2) taking h=0.1 $\frac{dy}{dx} = \frac{y-x}{y+x}$ , with y(0)=1	L4	CO 4

5	Using the Runge- Kutta method of order four, find y at x=0.2 given that $\frac{dy}{dx} = x + y$ , y=1 when x=0	L2	CO 4
6	Using the Runge- Kutta method of order 4, find y (0.2) given that $\frac{dy}{dx} = 3x + \frac{y}{2}$ , y (0)=1.	L3	CO 4
7	Apply the Runge- Kutta method to find y (0.2) if $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ , with y (0)=1.	L2	CO 4
8	Given $\frac{dy}{dx} = \frac{(1+x^2)y^2}{2}$ , $y(0) = 1$ , $y(0.1) = 1.06$ , $y(0.2) = 1.12$ , $y(0.3) = 1.21$ , evaluate $y(0.4)$ by using Milne's Predictor -Corrector method.	L3	CO 4
9	Applying Milne's Predictor –Corrector method ,to find y(1.4),from $\frac{dy}{dx} = x^2 + \frac{y}{2}$ , given that $y(1) = 2$ , $y(1.1) = 2.2156$ , $y(1.2) = 2.4549$ , $y(1.3) = 2.7514$	L2	CO 4
10	Using Mathematical tools, write the code to find the solution of $\frac{dy}{dx} = 1 + \frac{y}{x}$ at $y(2)$ taking h = 0.2. Given that $y(1) = 2$ by Runge-Kutta 4th order method.	L5	CO 4
11	Given $\frac{dy}{dx} = xy + y^2$ , y (0) = 1, y (0.1) = 1.1169, y (0.2) = 1.2773, y (0.3) = 1.5049 compute y(0.4) using Milne's method.	L2	CO 4
12	Employ Taylors series method to obtain approximate value of y at $x = 0.2$ for the given $\frac{dy}{dx} = x^2y - 1$ Find y for 5 decimal places of accuracy.	L2	CO 4
13	Using modified Euler's method find $y(0.1)$ , given $\frac{dy}{dx} = x^2 - y$ ; $y(0) = 1$ , Perform two iterations at each step, taking $h = 0.05$ .	L3	CO 4
14	Solve $(x + y)\frac{dy}{dx} = 1$ , $y(0.4) = 1$ using Runge-Kutta method of fourth order, hence find y at $x = 0.5$ and take $h = 0.1$	L3	CO 4

Note:

- 1. Questions shall be framed by consolidating comprehensively from the following sources
  - Exercise problems of text books/ references

- Previous year question VTU exam Question paper. (Mark the year/exam beside the question)
- Questions by Experts during Interview/Academic Audit
- Internet sources/ other Universities examination question papers.
- Own / experience.
- Gate questions mentioning the year.
- 2. Questions shall follow all the Bloom's learning levels with appropriate action verbs
- 3. There shall be a total of 50 questions considering10 questions from each module, of which, 3 questions each at L1 and L2, 2 questions at L3, 1 question each at L4 and L5/L6.
- 4. Ensure the coverage of all Cos.

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Sign. of the Faculty

Principal

HOD