# Question bank

## Module - 05

1) Employ Taylors series method to obtain approximate value of $y$ at $x=0.2$ for $\frac{dy}{dx} = 2y + 3e^x$, $y(0)=0$.

Given: $\frac{dy}{dx} = 2y + 3e^x$ — ①

$x_0 = 0$

$y(0) = 0$

$y' = 2y_0 + 3e^x$

$y(0) = 2(0) + 3e^{(0)}$

$\boxed{y'(0) = 3}$

$y'' = 2y' + 3e^x$

$y''(0) = 2(3) + 3e^{(0)}$

$y''(x_0) = 6+3$

$\boxed{y''(x_0 = 9}$

$y''' = 2y'' + 3e^x$

$y'''(x_0) = 2(9) + 3e^{(0)}$

$= 18+3$

$\boxed{y''(x_0) = 21}$

$$y(x) = y(x_0) + (x-x_0)y'(x_0) + \frac{(x-x_0)^2}{2!} y''(x_0) + \frac{(x-x_0)^3}{3!} y'''(x_0)$$

$$y(0.2) = 0 + (0.2-0)3 + \frac{(0.2-0)^2}{2}(9) + \frac{(0.2-0)^3}{3\times 2\times 1}(21)$$

$\boxed{y(0.2) = 0.808}$

$\therefore$ The value of $y$ at $x=0.2$ is $0.808$

2) Solve $\frac{dy}{dx} = e^x - y^2$ with the initial Condition $y(0)=1$ using taylor series method to evaluate $y(0.2)$

Given: $\frac{dy}{dx} = e^x - y^2$

$x_0 = 0$

$y_0 = 1$

$y' = e^x - y^2$

$y'(x_0) = e^{(0)} - (1)^2$

$\boxed{y'(x_0) = 0}$

$y'' = e^x - 2y \cdot y'$

$y''(x_0) = e^{(0)} - 2(y(0) \cdot y'(0))$

$= e^{(0)} - 2(1) \cdot (0)$

$\boxed{y''(x_0) = 1}$

$y''' = e^x - 2(y \cdot y'' + y'y')$

$y'''(0) = e^{(0)} - 2(y(0)\, y''(0) + y'(0)\, y'(0))$

$= e^0 - 2(1(1) + (0)(0)]$

$= 1 - 2$

$\boxed{y'''(0) = -1}$

$y^{IV} = e^x - 2(y \cdot y''' + y''y' + y'y'' + y'y'')$

$y^{IV}(0) = e^{(0)} - 2(1(-1) + 1(0) + (0)(1) + (0)(1))$

$= 1 - 2(-1)$

$= 1 + 2$

$\boxed{y^{IV}(0) = 3}$

$$y(x) = y(x_0) + (x - x_0)\, y'(x_0) + \frac{(x-x_0)^2}{2!} y''(x_0) + \frac{(x-x_0)^3}{3!} y'''(x_0) + \frac{(x-x_0)^4}{4!} y^{IV}(x_0)$$

$$= 1 + (0.2-0)0 + \frac{(0.2-0)^2}{2 \times 1}(1) + \frac{(0.2-0)^3}{3 \times 2 \times 1}(-1) + \frac{(0.2-0)^4}{4 \times 3 \times 2 \times 1}(3)$$

$\boxed{y(0.2) = 1.01866}$

$\therefore$ The value of $y$ at $x = 0.2$ is $1.01866$

3) By using modified Euler's method, obtain solution of the equation $\frac{dy}{dx} = x + |\sqrt{y}|$ with initial condition $y=1$ at $x=0$ for the range $0 \le x \le 0.4$ with step size of $0.2$

Given: $\frac{dy}{dx} = x + |\sqrt{y}| = f(x,y)$

$x_0 = 0$ , $y_0 = 1$ , $h = 0.2$

when $y >= 0$ $|\sqrt{y}| = \sqrt{y}$

Eulars Formula

at $x = 0.2$

$y_1^{(0)} = y_0 + h\, f(x_0, y_0)$

$= 1 + 0.2\,(x_0 + \sqrt{y_0})$

$= 1 + 0.2(0 + \sqrt{1})$

$\boxed{y_1^{(0)} = 1.2}$

From modified Eular formula

$y_1^{(1)} = y_0 + h/2 \left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$

$= 1 + \frac{0.2}{2} \left[1 + (x_1 + \sqrt{y_1^{(0)}})\right]$

$= 1 + 0.1 \left[1 + (0.2 + \sqrt{1.2})\right]$

$\boxed{y_1^{(1)} = 1.2295}$

$y_1^{(2)} = y_0 + h/2 \left(f(x_0, y_0) + f(x_1, y_1^{(1)})\right)$

$= 1 + \frac{0.2}{2} \left(1 + (0.2 + \sqrt{1.2295}\,)\right)$

$\boxed{y_1^{(2)} = 1.2308}$

hence $y(0.2) = 1.2308$

Eulars formula

at $x = 0.4$

$$y_1^{(0)} = y_0 + h\, f(x_0, y_0)$$

$$= 1 + 0.2\,(0 + \sqrt{1})$$

$$= 1 + 0.2\,(1)$$

$$\boxed{y_1^{(0)} = 1.2}$$

From modified Eular formula

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_0, y_1^{(0)})\right]$$

$$= 1 + \frac{0.2}{2}\left(1 + (x_1 + \sqrt{y_1^{(0)}}\right)$$

$$= 1 + 0.1\,(1 + (0.4 + \sqrt{1.2})$$

$$\boxed{y_1^{(1)} = 1.2495}$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left(f(x_0, y_0) + f(x_1, y_1^{(1)})\right)$$

$$= 1 + \frac{0.2}{2}\left(1 + (x_1 + \sqrt{y_1^{(1)}})\right)$$

$$= 1 + 0.1\left(1 + (0.4 + \sqrt{1.2495})\right)$$

$$\boxed{y_1^{(2)} = 1.2517}$$

4) By using modified Euler's method, Examine $y(0.2)$ taking $h = 0.1$ $\frac{dy}{dx} = \frac{y - x}{y + x}$, with $y(0) = 1$

Given:

$$\frac{dy}{dx} = \frac{y - x}{y + x}$$

$\frac{dy}{dx}$ comparing with $f(x, y)$

$x_0 = 0$ , $y_0 = 1$ , $h = 0.1$

$x_1 = x_0 + h = 0 + 0.1 = 0.1$

$x_2 = x_1 + h = 0.1 + 0.1 = 0.2$

From Eulers formula

$y_1^{(0)} = y_0 + h\, f(x_0, y_0)$

$= 1 + (0.1)\left[\frac{y_0 - x_0}{y_0 + x_0}\right]$

$= 1 + 0.1\left(\frac{1-0}{1+0}\right)$

$\boxed{y_1^{(0)} = 1.1}$

$y_1^{(1)} = y_0 + h/2\ (f(x_0, y_0) + f(x_1, y_1^{(0)}))$

$= 1 + 0.1/2\ \left(1 + \frac{y_1^{(0)} - x_1}{y_1^{(0)} + x_1}\right)$

$= 1 + 0.05\left(1 + \frac{1.1 - 0.1}{1.1 + 0.1}\right)$

$\boxed{y_1^{(1)} = 1.0916}$

$y_1^{(2)} = y_0 + h/2\ (f(x_0, y_0) + f(x_1, y_1^{(1)}))$

$= 1 + 0.1/2\ \left\{1 + \frac{y_1^{(1)} - x_1}{y_1^{(1)} + x_1}\right)$

$= 1.05\left(1 + \frac{1.0916 - 0.1}{1.0916 + 0.1}\right)$

$\boxed{y_1^{(2)} = 1.0916}$

when $x = 0.2$

From Eulars formula

$y_2^{(0)} = y_1 + h\, f(x_1, y_1)$

$= 1.0916 + 0.1\left[\frac{y_1 - x_1}{y_1 + x_1}\right] \Rightarrow 1.0916 + 0.1\left[\frac{1.0916 - 0.1}{1.0916 + 0.1}\right]$ $\boxed{y_2^{(0)} = 1.174}$

From modified Eulars Formula $f(x_1, y_1) = 0.8321$

$$y_2^{(1)} = y_1 + h/2\,(f(x_1,y_1) + f(x_2, y_2^{(0)}))$$

$$= 1.0916 + \frac{0.1}{2}\left[0.8321 + \left[\frac{1.09174 - 0.2}{1.174 + 0.2}\right]\right]$$

$$\boxed{y_2^{(1)} = 1.1686}$$

$$y_2^{(2)} = y_1 + h/2\,[f(x_1,y_1) + f(x_1, y_2^{(1)})]$$

$$= 1.0916 + \frac{0.1}{2}\left[0.8321 + \left[\frac{y_2^{(1)} - x_2}{y_2^{(1)} + x_2}\right]\right]$$

$$= 1.0916 + 0.05\left[0.8321 + \frac{1.1686 - 0.2}{1.1686 + 0.2}\right]$$

$$\boxed{y_2^{(2)} = 1.1686}$$

5) Using the Runge-kutta method of order four, find $y$ at $x = 0.2$ given that $\frac{dy}{dx} = x + y$, $y = 1$ when $x = 0$

Given: $\frac{dy}{dx} = x + y$

$x_0 = 0$ $\quad y_0 = 1$ $\quad h = 0.2$

$$\frac{dy}{dx} = f(x, y) = x + y$$

$k_1 = h\,f(x_0, y_0)$

$k_1 = 0.2\,(x_0 + y_0)$

$k_1 = 0.2\,(0 + 1)$

$\boxed{k_1 = 0.2}$

$k_2 = h\,f(x_0 + h/2,\ y_0 + k_1/2)$

$k_2 = 0.2\,f\left(0 + \frac{0.2}{2},\ 1 + \frac{0.2}{2}\right)$

$= 0.2\,f(0.1,\ 1.1)$

$= 0.2\,(x_1 + y_1)$

$k_2 = 0.2\,(0.1 + 1.1)$

$\boxed{k_2 = 0.24}$

$K_3 = h\,f(x_0 + h/2,\ y_0 + K_2/2)$

$= 0.2\ f(0 + 0.2/2,\ 1 + \frac{0.24}{2})$

$= 0.2\ f(0.1,\ 1.12)$

$K_3 = 0.2\ (0.1 + 1.12)$

$\boxed{K_3 = 0.244}$

$K_4 = h\,f(x_0 + h,\ y_0 + K_3)$

$= 0.2\ f(0 + 0.2,\ 1 + 0.244)$

$= 0.2\ f(0.2,\ 1.244)$

$= 0.2\ [0.2 + 1.244]$

$\boxed{K_4 = 0.2888}$

From R K method

$y_1 = y_0 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= 1 + 1/6\ [0.2 + 2(0.24) + 2(0.244) + 0.2888)$

$= 1 + \frac{1}{6}\ [1.4568)$

$\boxed{y_1 = 1.2428}$

$\therefore$ Value of $y$ at $x = 0.2$ is $1.2428$

6) Using the Runge Kutta method of order 4, find $y(0.2)$ given that $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y(0) = 1$

Given: $\frac{dy}{dx} = 3x + y/2$

$f(x,y) = 3x + y/2$

$x_0 = 0 \qquad y_0 = 1 \qquad h = 0.2$

$K_1 = h\,f(x_0, y_0)$

$= 0.2\ (3x_0 + y_0/2)$

$= 0.2\ [3(0) + 1/2]$

$\boxed{K_1 = 0.1}$

$K_2 = h\ f(x_0 + h/2,\ y_0 + K_1/2)$

$= 0.2\ f(0 + 0.2/2,\ 1 + 0.1/2]$

$= 0.2\ f(0.1,\ 1.05)$

$= 0.2\ [3(0.1) + 1.05/2]$

$\boxed{K_2 = 0.165}$

$K_3 = hf(x_0 + h/2, y_0 + K_2/2)$

$= 0.2\, f(0 + \frac{0.2}{2}, 1 + \frac{0.165}{2})$

$= 0.2\, f(0.1, 1.0825)$

$K_3 = 0.2\, [3(0.1) + \frac{1.0825}{2})$

$\boxed{K_2 = 0.16825}$

$K_4 = hf(x_0 + h, y_0 + K_3)$

$= 0.2\, f(0 + 0.2, 1 + 0.16825)$

$= 0.2\, f(0.2, 1.16825)$

$K_4 = 0.2\, [3(0.2) + \frac{1.16825}{2})$

$\boxed{K_4 = 0.2368}$

From R K method

$y_1 = y_0 + 1/6\, [K_1 + 2K_2 + 2K_3 + K_4]$

$= 1 + 1/6\, [0.1 + 2(0.165) + 2(0.16825) + 0.2368)$

$\boxed{y_1 = 1.1672}$

$y(0.2) \approx 1.1672$

7) Apply the Runge-kutta method to find $y(0.2)$ if $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$

Given: $f(x, y) = \frac{y^2 - x^2}{y^2 + x^2}$

$x_0 = 0, y_0 = 1 \quad h = 0.2$

$K_1 = h\, f(x_0, y_0)$

$= 0.2\, \left[\frac{(1)^2 - 0}{(1)^2 + 0}\right)$

$\boxed{K_1 = 0.2}$

$K_2 = hf(x_0 + h/2, y_0 + K_1/2)$

$= 0.2\, f\left(0 + \frac{0.2}{2}, 1 + \frac{0.2}{2}\right]$

$= 0.2\, f(0.1, 1.1]$

$K_2 = 0.2 \left[\frac{(1.1)^2 - (0.1)^2}{(1.1)^2 + (0.1)^2}\right]$

$\boxed{K_2 = 0.9836}$

$K_3 = h\,f(x_0+h/2,\ y_0+K_2/2)$

$K_3 = 0.2\,f\left(0+\frac{0.2}{2},\ 1+\frac{0.9836}{2}\right]$

$K_3 = 0.2\,f\left(0.1,\ 1.4918\right]$

$K_3 = 0.2\left[\frac{(1.4918)^2-(0.1)^2}{(1.4918)^2+(0.1)^2}\right]$

$\boxed{K_3 = 0.1982}$

$K_4 = h\,f(x_0+h,\ y_0+K_3)$

$= 0.2\ f(0+0.2\ ,\ 1+0.1982)$

$= 0.2\ f(0.2\ ,\ 1.1982)$

$K_4 = 0.2\left[\frac{(1.1982)^2-(0.2)^2}{(1.1982)^2+(0.2)^2}\right]$

$\boxed{K_4 = 0.1959}$

From R K method

$y_1 = y_0 + 1/6\,[K_1+2K_2+2K_3+K_4]$

$= 1+\frac{1}{6}\,[0.2+2(0.9836)+2(0.1982)+0.1959]$

$\boxed{y_1 = 1.4599}$

$y(0.2)$ ~~is~~ $1.4599$

8) Given $\frac{dy}{dx} = \frac{(1+x^2)y^2}{2}$ $y(0)=1$, $y(0.1)=1.06$, $y(0.2)=1.12$, $y(0.3)=1.21$, evaluate $y(0.4)$ by using milne's predictor-Corrector method.

| $x$ | $y$ | $y' = \frac{(1+x^2)y^2}{2}$ |
|---|---|---|
| 0 | 1 | 0.5 |
| 0.1 | 1.06 | 0.5674 |
| 0.2 | 1.12 | 0.6522 |
| 0.3 | 1.21 | 0.7979 |
| 0.4 | 1.2764 | 0.9449 |

$y_4^p = y_0 + \frac{4h}{3}\,(2y_1' - y_2' + 2y_3')$

$= 1 + \frac{4(0.1)}{3}\,(2(0.5674) - 0.6522 + 2(0.7979))$

$y_4^p = \cancel{1.0644}\ 1.2764$

$y_4^c = y_2 + h/3\,(y_2' + 4y_3' + y_4')$

$= 1.12 + \frac{0.1}{3}\,(0.6522 + 4(0.7979) + 0.9375)$

$= 1.12 + 0.0333\,(4.7813)$

$y_4^c = 1.2792$

y at x (0.4) is 1.2792

9) Applying milnes predictor - corrector method, to find $y(1.4)$. From $\frac{dy}{dx} = x^2 + \frac{y}{2}$, given that $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4549$, $y(1.3) = 2.7514$

Given, $\frac{dy}{dx} = x^2 + y/2$

| $x$ | $y$ | $y' = x^2 + y/2$ |
|---|---|---|
| 1 | 2 | 2 |
| 1.1 | 2.2156 | 2.3178 |
| 1.2 | 2.4549 | 2.6674 |
| 1.3 | 2.7514 | 3.0657 |
| 1.4 | 3.0796 | 3.4998 |

From milne's predictor formula

$y_1^p = y_0 + \frac{4h}{3}\cdot(2y_1' - y_2' + 2y_3')$

$= 2 + \frac{4(0.1)}{3}\,(2(2.3178) - 2.6674 + 2(3.0657))$

$= 2 + 0.1333\,(8.0996)$

$y_1^p = 3.0796$

from milnes Corrector formula

$y_4^c = y_2 + h/3 \, (y_2' + 4y_3' + y_4')$

$= 2.4549 + \frac{0.1}{3} [2.6674 + 4(3.0657) + 3.4998)$

$\boxed{y_4^c = 3.0692}$

$y(1.4)$ is $3.0692$

10) Apply the Range kutta method to find the solution of $dy/dx = 1 + (y/x)$ at $y(2)$ taking $h = 0.2$, given that $y(1) = 2$.

```
from sympy import *
import numpy as np
def Rungekutta (g, xo, h, yo, xn):
    x, y = symbols ('x, y')
    f = lambdify ([x, y], g)
    xt = xo + h
    Y = [yo]
    while xt <= xn:
        k1 = h*f (xo, yo)
        k2 = h*f (xo + h/2, yo + k1/2)
        k3 = h*f (xo+h/2, yo+k2/2)
        k4 = h*f (xo+h, yo+k3)
    y1 = yo+(1/6)* (k1+ 2*k2 + 2* k3 + k4)
```

```
        y.append (y1)
        x0 = xt
        y0 = y1
        xt = xt + h
    return np.round (y, 2)
Rungekutta (1 + (y/x)', 1, 0.2, 2, 2)
```

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11) Given $dy/dx = xy + y^2$, $y(0) = 1$, $y(0.1) = 1.1169$, $y(0.2) = 1.2773$, $y(0.3) = 1.5049$, Compute $y$ at 0.4 by using milnes method.

| $x$ | $y$ | $y' = xy + y^2$ |
|---|---|---|
| 0 | 1 | 1 |
| 0.1 | 1.1169 | 1.3591 |
| 0.2 | 1.2773 | 1.8869 |
| 0.3 | 1.5049 | 2.7161 |
| 0.4 | 1.8351 | 4.1016 |

from milnes predictor method

$$y_4^c = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3')$$

$$= 1 + \frac{4(0.1)}{3}(2(1.3591) - 1.8869 + 2(2.7161))$$

$$y_4^c = 1.8351$$

$y_4^c = y_2 + h/3\,(y_2' + 4y_3' + y_4')$

$= 1.0773 + \frac{0.1}{3}(1.8869 + 4(2.7161) + 4.1016)$

$y_4^c = 1.8390$

$y$ at $0.4$ is $1.8390$

18) Employ Taylors series method to obtain approximate value of $y$ at $x = 0.2$ for the given $dy/dx = x^2y - 1$. Find $y$ for 5 decimal places of accuracy.

Given: $\frac{dy}{dx} = x^2y - 1$

Consider $y(0) = 1$

$y'(x_0) = (0)^2(1) - 1$

$\boxed{y'(x_0) = -1}$

$y''(x_0) = x^2y' + y2(x) - 0$

$= 0(-1) + (1)2(0) - 0$

$\boxed{y''(x_0) = 0}$

$y'''(x_0) = 2xy' + y''x^2 + 2(y(1) + xy')$

$= 2(0)(-1) + (0)(0) + 2(1(1) + (0)(-1))$

$\boxed{y'''(x_0) = 2}$

$y(x) = y(x_0) + (x - x_0)\,y'(x_0) + \frac{(x-x_0)^2}{2!}\,y''(x_0) + \frac{(x-x_0)^3}{3!}\,y'''(x_0)$

$= 1 + (0.2 - 0)(-1) + \frac{(0.2-0)^2}{2}(0) + \frac{(0.2-0)}{6}(2)$

$\boxed{y(0.2) = 0.8026}$

14) solve $(x+y)\frac{dy}{dx} = 1$, $y(0.4) = 1$ using Runge kutta method of fourth order. hence find y at $x = 0.5$ and take $h = 0.1$

Given: $dy/dx = \frac{1}{x+y}$

$f(x,y) = \frac{1}{x+y}$

$x_0 = 0.4$

$y_0 = 1$

$k_1 = h\,f(x_0, y_0)$

$= 0.1\left(\frac{1}{x_0+y_0}\right)$

$= 0.1\left(\frac{1}{0.4+1}\right)$

$\boxed{k_1 = 0.0714}$

$k_2 = h\,f(x_0+h/2,\ y_0+k_1/2)$

$= 0.1\,f\left(0.4+0.1/2,\ 1+\frac{0.0714}{2}\right)$

$= 0.1\ f(0.5,\ 1.0357)$

$\boxed{k_2 = 0.0673}$

$k_3 = h\,f(x_0+h/2,\ y_0+k_2/2)$

$k_3 = 0.1\,f\left(0.4+0.1/2,\ 1+\frac{0.0673}{2}\right)$

$k_3 = 0.1\,f(0.45,\ 1.03365)$

$k_3 = 0.1\left(\frac{1}{0.45+1.03365}\right)$

$\boxed{k_3 = 0.0674}$

$k_4 = h\,f(x_0+h,\ y_0+k_3)$

$= 0.1\,f(0.4+0.1,\ 1+0.0674)$

$= 0.1\,f(0.5,\ 1.0674)$

$= 0.1\left(\frac{1}{0.5+1.0674}\right)$

$\boxed{k_4 = 0.06379}$

$y_1 = y_0 + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$

$= 1 + \frac{1}{6}\left[0.0714 + 2(0.0673) + 2(0.0674) + 0.06379\right)$

$y_1 = 1.0674$

$\boxed{y(0.5) = 1.0674}$